PHASE TRANSFORM ALGORITHM FOR RADIO OCCULTATION DATA PROCESSING

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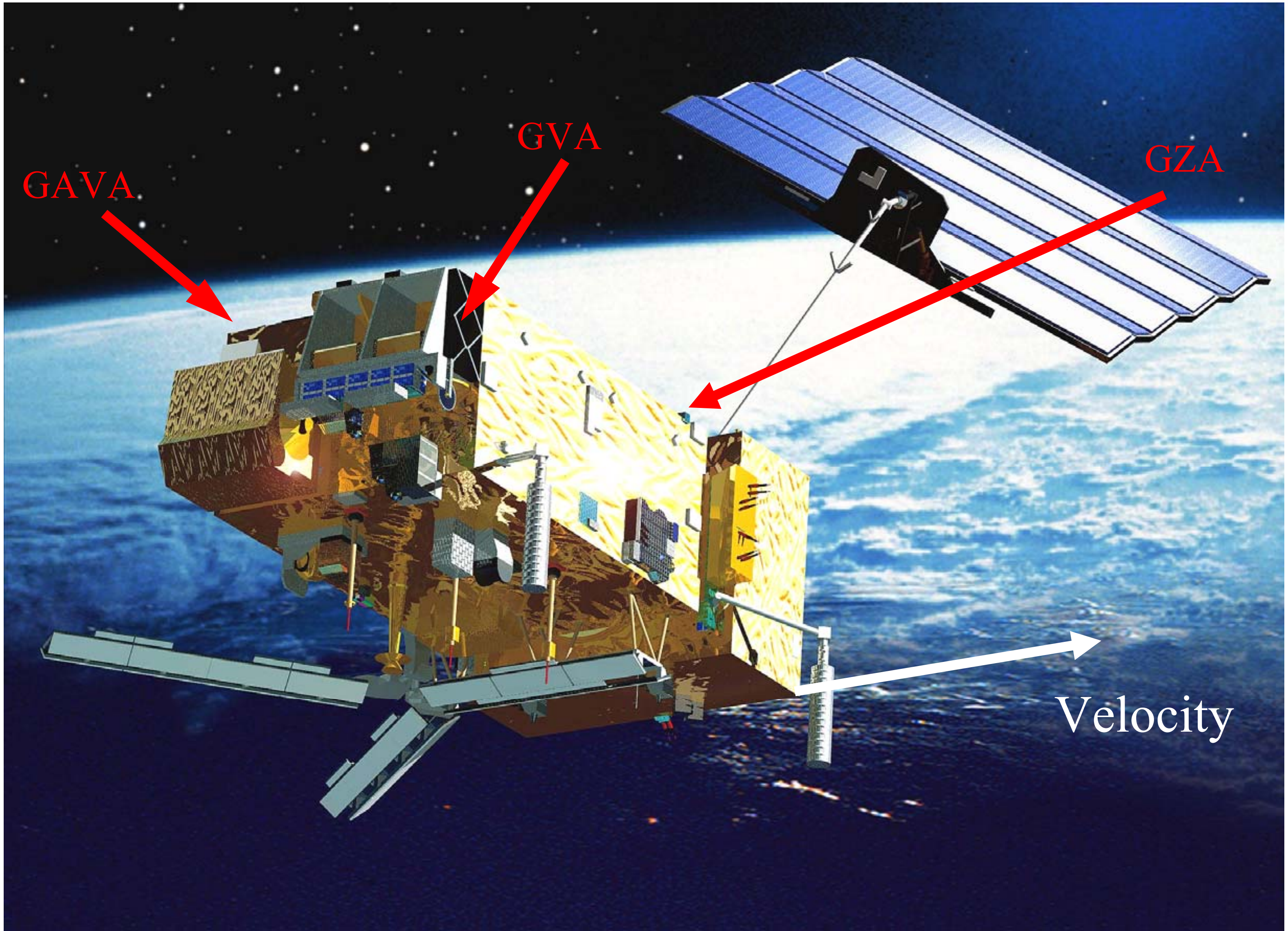
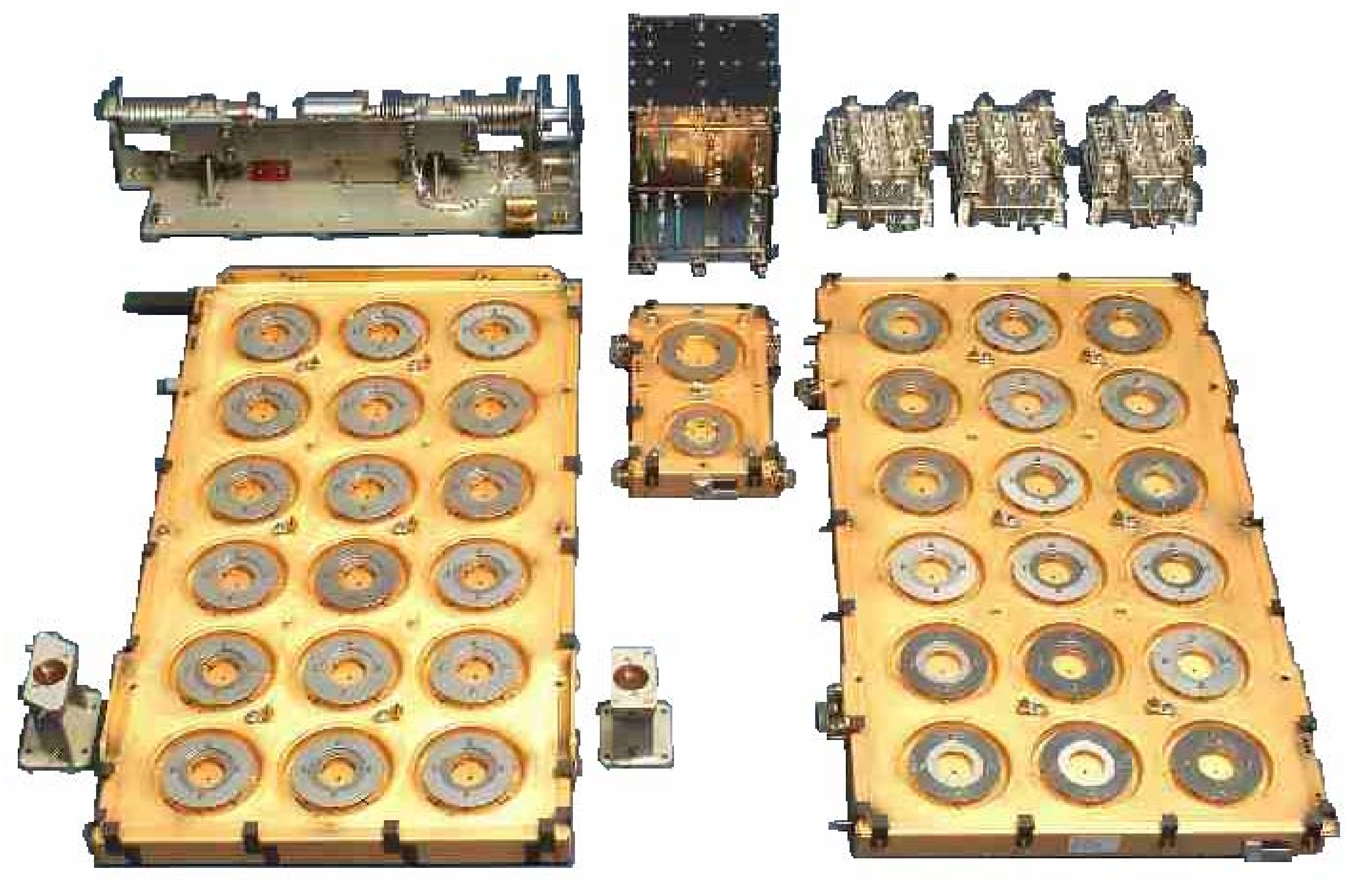
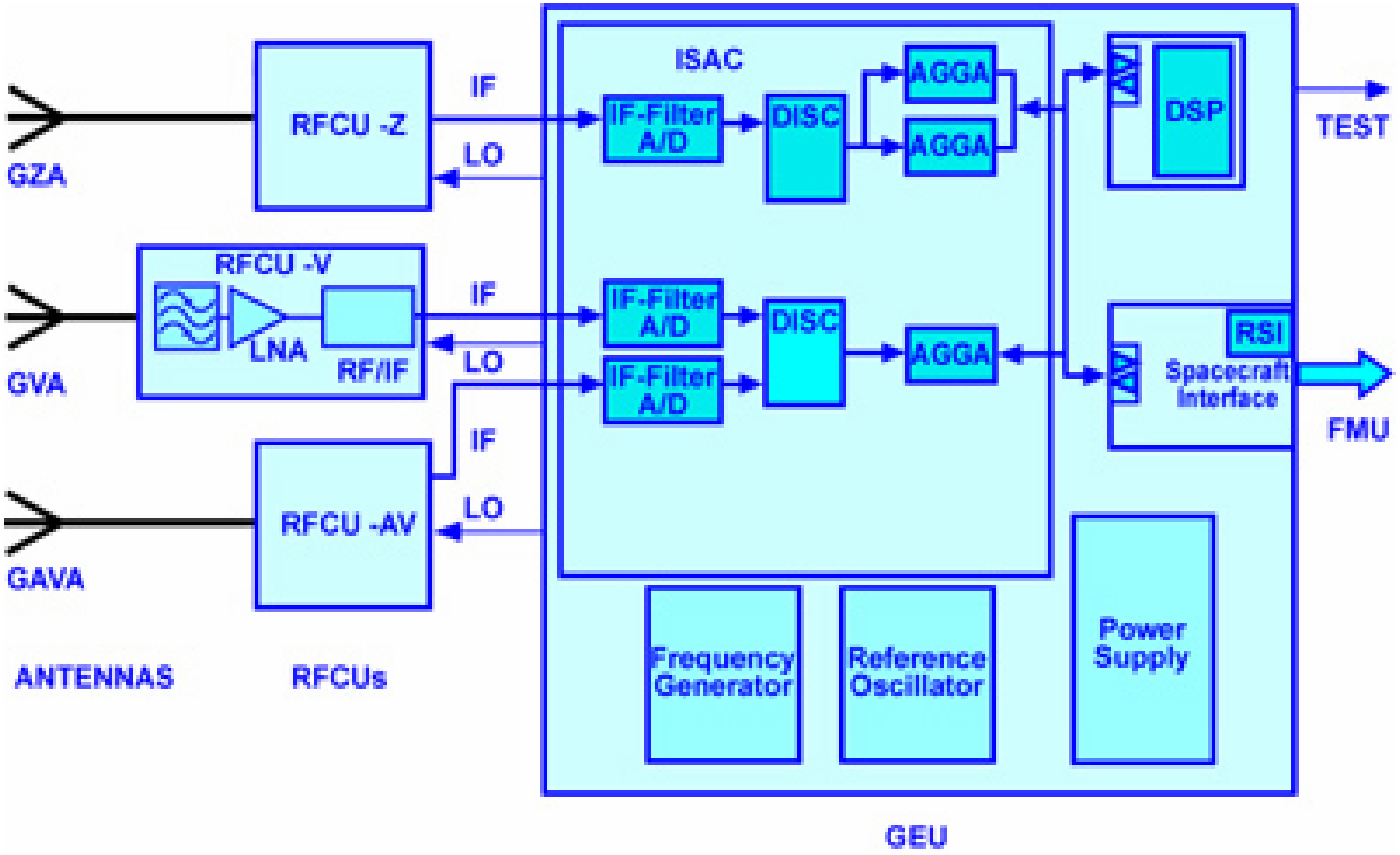
GRAS ON A METOP SATELLITE



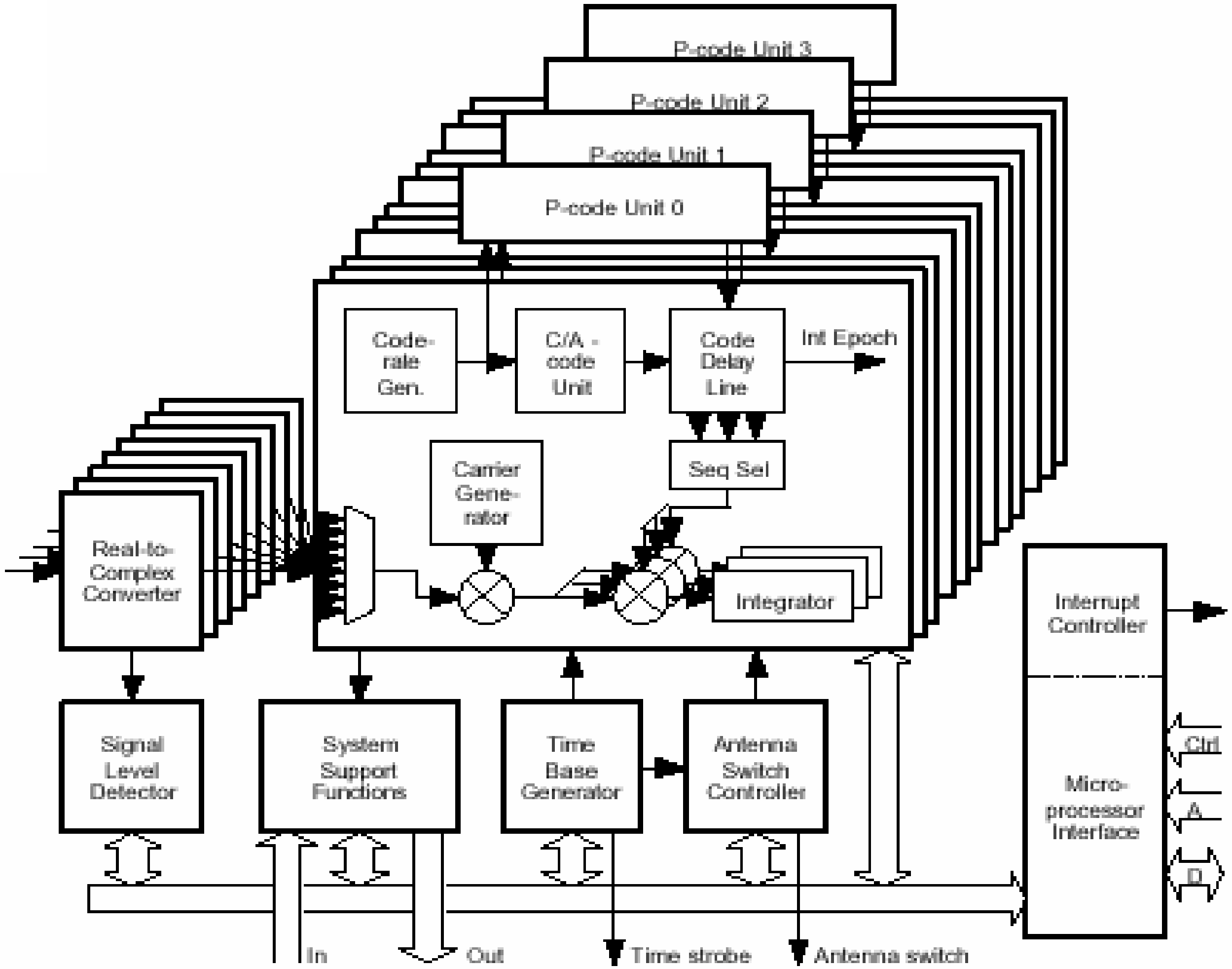
GRAS INSTRUMENT HARDWARE



GRAS INSTRUMENT ARCHITECTURE



AGGA ASIC ARCHITECTURE



GRAS Measurements

L1 C/A Code Phase
L1 P Code Phase
L1 Carrier Phase
L1 Signal Amplitude

L2 P Code Phase
L2 Carrier Phase
L2 Signal Amplitude

All time stamped with Instrument Measurement Time (IMT)

The regenerated carrier phase (on L1 or L2 channels) is obtained by adding back the signals removed by fixed down conversions and by the tracking loop or law-based final down conversion:

$$\left\{ \left(\varphi_{REG}(i), \tau_{RX}^{IMT}(i) \right); i : 1 \rightarrow N \right\}$$

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The Regenerated Phase

$$\varphi_{REG}\left(\tau_{RX}^{IMT}\right) =$$

$$\frac{2\pi f_{GPS(n)} \left| \mathbf{r}_{GRAS-ANT(m)}^{ECI}\left(t_{RX}^{REF}\right) - \mathbf{r}_{GPS(n)-ANT}^{ECI}\left(t_{TX}^{REF}\right) \right|}{c}$$

$$+$$

$$2\pi f_{GPS(n)} \; G_{GREL-TOF}\left[\mathbf{r}_{GRAS-ANT(m)}^{ECI}\left(t_{RX}^{REF}\right), \mathbf{r}_{GPS(n)-ANT}^{ECI}\left(t_{TX}^{REF}\right) \right]$$

$$+$$

$$2\pi f_{GPS(n)} \left(\Delta_{IMT}\left(t_{RX}^{REF}\right) - \Delta_{GPS(n)}\left(t_{TX}^{REF}\right) \right)$$

$$+$$

$$\varphi_{TROP}\left(\tau_{RX}^{IMT}\right) - \varphi_{IONO}\left(\tau_{RX}^{IMT}\right)$$

$$+$$

$$\varphi_{CH(i)}\left(\theta\left(t_{RX}^{REF}\right), \phi\left(t_{RX}^{REF}\right), T_k\left(t_{RX}^{REF}\right) \right)$$

$$+$$

$$N_{CS}\left(\tau_{RX}^{IMT}\right) + \varphi_{CONSTANT}$$

The Correction of the Regenerated Phase I

$$\varphi_{REG\text{-}0}\left(\tau_{RX}^{IMT}\right) = \varphi_{REG}\left(\tau_{RX}^{IMT}\right)$$

$$-$$

$$\Lambda_{CH}\left(\theta\left(t_{RX}^{APPROX}\right), \phi\left(t_{RX}^{APPROX}\right), T_{ANT(m)}\left(t_{RX}^{APPROX}\right)\right)$$

$$-$$

$$\Xi_{CH}\left(T_{RFCU(m)}\left(t_{RX}^{APPROX}\right), g_{ANA\text{-}RFCU(m)}\left(t_{RX}^{APPROX}\right)\right)$$

$$-$$

$$\Gamma_{CH}\left(T_{GEU}\left(t_{RX}^{APPROX}\right)\right)$$

The Correction of the Regenerated Phase II

$$\varphi_{\text{REG-A}}\left(\tau_{\text{RX}}^{\text{IMT}}\right) =$$
$$\varphi_{\text{REG-0}}\left(\tau_{\text{RX}}^{\text{IMT}}\right) -$$
$$2\pi f_{\text{GPS(n)}}\ G_{\text{GREL-TOF}}\left[\mathbf{r}_{\text{GRAS-ANT(m)}}^{\text{ECI}}\left(t_{\text{RX}}^{\text{REF}}\right),\ \mathbf{r}_{\text{GPS(n)-ANT}}^{\text{ECI}}\left(t_{\text{TX}}^{\text{REF}}\right)\right]$$

$$\varphi_{\text{REG-B}}\left(\tau_{\text{RX}}^{\text{IMT}}\right) = \varphi_{\text{REG-A}}\left(\tau_{\text{RX}}^{\text{IMT}}\right) -$$
$$2\pi f_{\text{GPS(n)}}\left(\Delta_{\text{IMT}}\left(t_{\text{RX}}^{\text{REF}}\right) - \Delta_{\text{GPS(n)}}\left(t_{\text{TX}}^{\text{REF}}\right)\right)$$

The Phase Transform Algorithm

$$R_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right) = \mathrm{Exp}\left[+i\ \varphi_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right)\right]$$

$$u(p) = \int_{0}^{T_{IMT}} \omega\left(\tau_{RX}^{IMT}\right) R_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right) \mathrm{Exp}\left[-i\ \theta\left(p, \tau_{RX}^{IMT}\right)\right] d\tau_{RX}^{IMT}$$

$$\alpha(p) = -\frac{\lambda}{2\pi}\frac{d}{dp}\left[\mathrm{Phase}\left[u(p)\right]\right]$$

$$\alpha_N(p) = \alpha_{L1}(p) + \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2}\left(\alpha_{L1}(p) - \alpha_{L2}(p)\right)$$

The Reference Phase Function

$$\theta(p,\tau) = +\frac{2\pi}{\lambda}\left[\begin{array}{l} \sqrt{r_{LEO}^2(\tau)-p^2} \quad + \quad \sqrt{r_{GPS}^2(\tau)-p^2} \quad + \quad p\,\Gamma(\tau) \\ -\,p\,\mathrm{Arctan}\left[\dfrac{\sqrt{r_{LEO}^2(\tau)-p^2}}{p}\right] \\ -\,p\,\mathrm{Arctan}\left[\dfrac{\sqrt{r_{GPS}^2(\tau)-p^2}}{p}\right] \end{array}\right]$$

$$\Gamma(\tau) = \mathrm{Arccos}\left[\frac{r_{LEO}(\tau)\,.\,r_{GPS}(\tau)}{|\,r_{LEO}(\tau)\,|\;|\,r_{GPS}(\tau)\,|}\right]$$

Derivation of Phase Transform Algorithm I

$$\varphi = \frac{2\pi}{\lambda} \int_{s} n(s)\,ds$$

$$\varphi = \frac{2\pi}{\lambda} \int_{s} n(r) \sqrt{1 + r^2 \left(\frac{d\theta}{dr} \right)^2}\, dr$$

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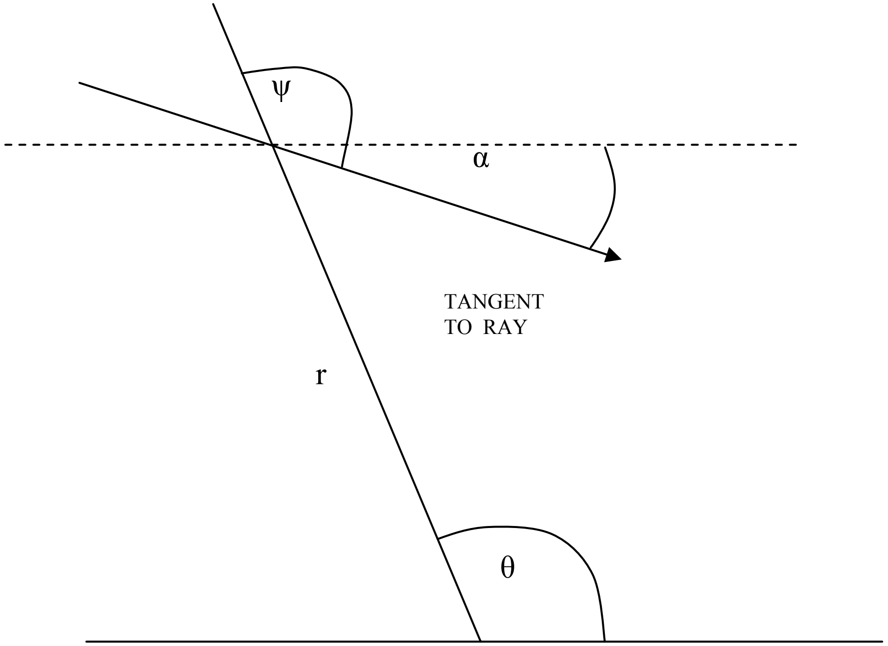
Derivation of Phase Transform Algorithm II



$$ds = r\,\mathrm{cosec}(\psi)\,d\theta$$

$$ds = \sqrt{(dr)^2 + r^2 (d\theta)^2}$$

$$a = n(r)\,r\sin(\psi)$$ The Formula of Bouguer

Derivation of the Phase Transform Algorithm III

$$\left(\frac{d\theta}{dr}\right)^2 = \frac{a^2}{r^2\left(n^2(r)\,r^2 - a^2\right)}$$

$$\varphi = \frac{2\pi}{\lambda}\int_{r_1}^{r_2}\frac{n^2(r)\,r\,dr}{\sqrt{n^2(r)\,r^2 - a^2}}$$

$$\varphi = \frac{2\pi}{\lambda}\int_{r_0}^{r_1}\frac{n^2(r)\,r\,dr}{\sqrt{n^2(r)\,r^2 - a^2}} + \frac{2\pi}{\lambda}\int_{r_0}^{r_2}\frac{n^2(r)\,r\,dr}{\sqrt{n^2(r)\,r^2 - a^2}}$$

Derivation of Phase Transform Algorithm IV

$$n^2 r = \frac{1}{2}\frac{d}{dr}\left(n^2 r^2\right) - \left(n^2 r^2 - a^2\right)\frac{1}{n}\frac{dn}{dr} - a^2 \frac{1}{n}\frac{dn}{dr}$$

$$n^2\left(r_0\right) r_0^{\,2} = a^2$$

$$\varphi_{OA} = \frac{2\pi}{\lambda}\sqrt{n^2\left(r_A\right) r_A^{\,2} - a^2}$$

$$- \frac{2\pi}{\lambda}\int_{r_0}^{r_A} \sqrt{n^2(r)\, r^2 - a^2}\,\frac{d\left(\ln(n)\right)}{dr}\, dr$$

$$+ \frac{2\pi a}{\lambda}\int_{r_0}^{r_A} \frac{-a}{\sqrt{n^2(r)\, r^2 - a^2}}\,\frac{d\left(\ln(n)\right)}{dr}\, dr$$

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Derivation of Phase Transform Algorithm V

$$\alpha = \psi - \theta \qquad \frac{d\alpha}{dr} = \frac{d\psi}{dr} - \frac{d\theta}{dr}$$

$$\frac{d\psi}{dr} = -\frac{a}{\sqrt{n^2(r)\, r^2 - a^2}}\frac{1}{n}\frac{d(n)}{dr} - \frac{a}{\sqrt{n^2(r)\, r^2 - a^2}}\frac{1}{r}$$

$$\frac{d\theta}{dr} = -\frac{a}{\sqrt{n^2(r)\, r^2 - a^2}}\frac{1}{r}$$

$$\frac{d\alpha}{dr} = -\frac{a}{\sqrt{n^2(r)\, r^2 - a^2}}\frac{d(\ln(n))}{dr}$$

Derivation of Phase Transform Algorithm VI

$$\varphi_{OA} = \frac{2\pi}{\lambda}\sqrt{n^2(r_A)\, r_A{}^2 - a^2}$$

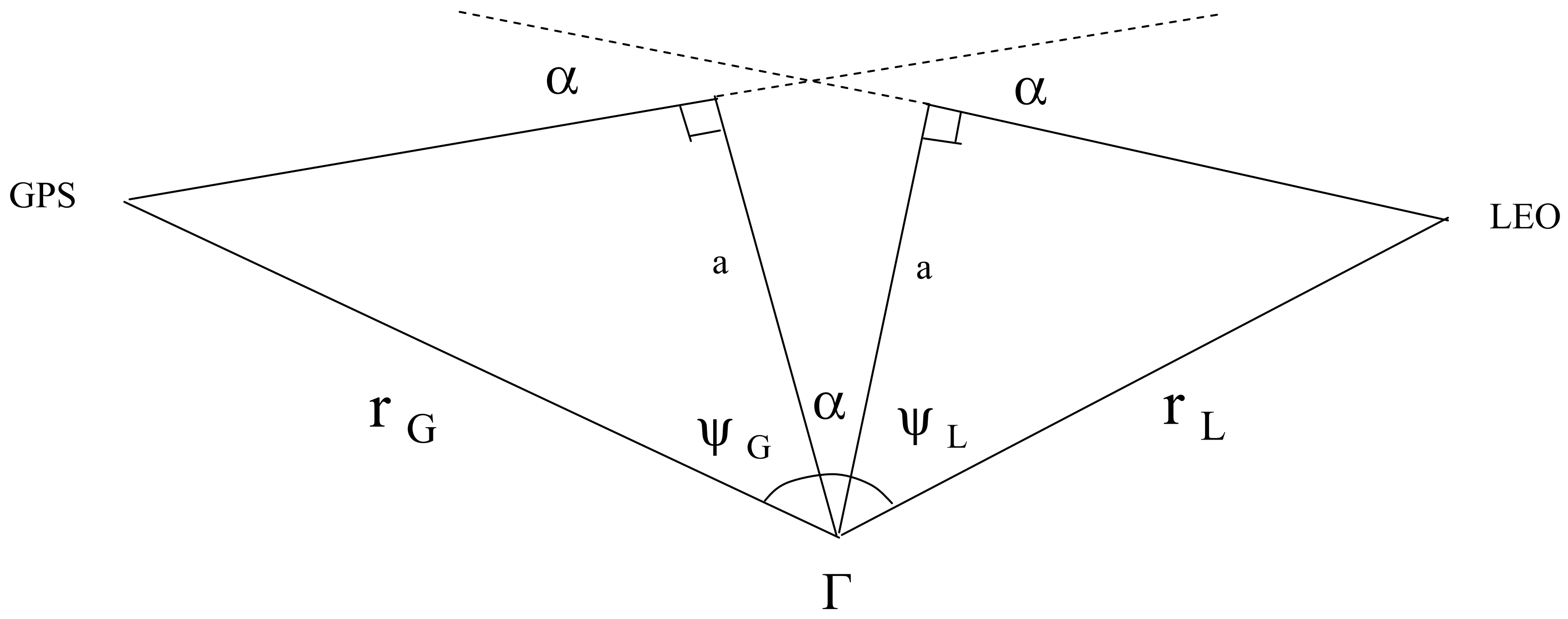
$$- \frac{2\pi}{\lambda}\int_{r_0}^{r_A}\sqrt{n^2(r)\, r^2 - a^2}\,\frac{d\left(\ln(n)\right)}{d\,r}\,d\,r$$

$$+ \frac{2\pi a}{\lambda}\,\alpha_{A0}$$

Derivation of Phase Transform Algorithm VII

$$\varphi = \frac{2\pi}{\lambda}\left[\sqrt{r_{LEO}^{2} - a^{2}} + \sqrt{r_{GPS}^{2} - a^{2}} - 2 \int_{r_0}^{\infty} \sqrt{n^{2}(r)\, r^{2} - a^{2}}\, \frac{d(\ln(n))}{dr}\, dr + a\,\alpha \right]$$

Derivation of Phase Transform Algorithm VIII



Derivation of Phase Transform Algorithm IX

$$\alpha = \Gamma - \psi_G - \psi_L$$

$$\alpha = \mathrm{Arccos}\left[\frac{\mathbf{r}_L \cdot \mathbf{r}_G}{|\mathbf{r}_L| \, |\mathbf{r}_G|} \right] - \mathrm{ArcTan}\left[\frac{\sqrt{r_G^2 - a^2}}{a} \right] - \mathrm{ArcTan}\left[\frac{\sqrt{r_L^2 - a^2}}{a} \right]$$

Derivation of Phase Transform Algorithm X

Now consider the integral:

$$u(p) = \int_0^{T_{IMT}} \omega\left(\tau_{RX}^{IMT}\right) R_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right) \mathrm{Exp}\left[-i\ \theta\left(p, \tau_{RX}^{IMT}\right)\right] d\tau_{RX}^{IMT}$$

$$u(p) = \int_0^{T_{IMT}} \omega\left(\tau_{RX}^{IMT}\right) A_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right) \mathrm{Exp}\left[i\ \varphi\left(p, \tau_{RX}^{IMT}\right) - i\ \theta\left(p, \tau_{RX}^{IMT}\right)\right] d\tau_{RX}^{IMT}$$

Since:

$$R_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right) = A_{REG\text{-}B}\left(\tau_{RX}^{IMT}\right) \mathrm{Exp}\left[+i\ \varphi\left(p, \tau_{RX}^{IMT}\right)\right]$$

Derivation of Phase Transform Algorithm XI

Integrating by the method of Stationary Phase yields:

$$u(p) = \frac{\left(\sqrt{2\pi i}\ \omega\left(\tau_{\text{RX-1}}^{\text{IMT}}\right) A_{\text{REG-B}}\left(\tau_{\text{RX-1}}^{\text{IMT}}\right).\ \text{Exp}\left[i\ \varphi\left(p, \tau_{\text{RX-1}}^{\text{IMT}}\right) - i\ \theta\left(p, \tau_{\text{RX-1}}^{\text{IMT}}\right) \right] \right)}{\sqrt{\left(\frac{d^2\varphi}{d\tau^2} - \frac{d^2\theta}{d\tau^2} \right)\Bigg|_{\tau = \tau_1}}}$$

Derivation of Phase Transform Algorithm XII

The function θ may be arbitrarily selected and therefore may be chosen to be:

$$\theta(p,\tau) = \frac{2\pi}{\lambda}\left[\sqrt{r_{LEO}(\tau)^2 - p^2} + \sqrt{r_{GPS}(\tau)^2 - p^2} + p\,\alpha(p,\tau) \right]$$

Where:

$$\alpha(p,\tau) = \Gamma(\tau) - \mathrm{ArcTan}\left[\frac{\sqrt{r_G^2(\tau) - p^2}}{p}\right] - \mathrm{ArcTan}\left[\frac{\sqrt{r_L^2(\tau) - p^2}}{p}\right]$$

Derivation of Phase Transform Algorithm XIII

With this choice of θ the phase of the function u becomes:

$$\text{Phase}\left[\, u\left(p \right) \right] = \left[\, \varphi\left(p, \tau_1 \right) - \theta\left(p, \tau_1 \right) \right]$$

$$= -\frac{4\pi}{\lambda} \int_{r_0}^{\infty} \sqrt{n^2(r)\, r^2 - p^2}\; \frac{d\left(\ln\left(n(r) \right) \right)}{dr}\, dr$$

Derivation of Phase Transform Algorithm XIV

Now consider the derivative of the phase of u with respect to p:

$$\frac{d}{dp}\left[\text{Phase}\left[u(p) \right] \right] = -\frac{4\pi}{\lambda}\frac{d}{dp}\int_{r_0}^{\infty} \sqrt{n^2(r)\, r^2 - p^2}\, \frac{d\left(\ln(n(r))\right)}{dr}\, dr$$

$$= -\frac{4\pi}{\lambda}\int_{r_0}^{\infty} -\frac{p}{\sqrt{n^2(r)\, r^2 - p^2}}\, \frac{d\left(\ln(n(r))\right)}{dr}\, dr$$

$$= -\frac{4\pi}{\lambda}\int_{r_0}^{\infty} \frac{d\alpha}{dr}\, dr$$

$$= -\frac{4\pi}{\lambda}\left(\frac{\alpha}{2}\right) = -\frac{2\pi}{\lambda}\alpha$$