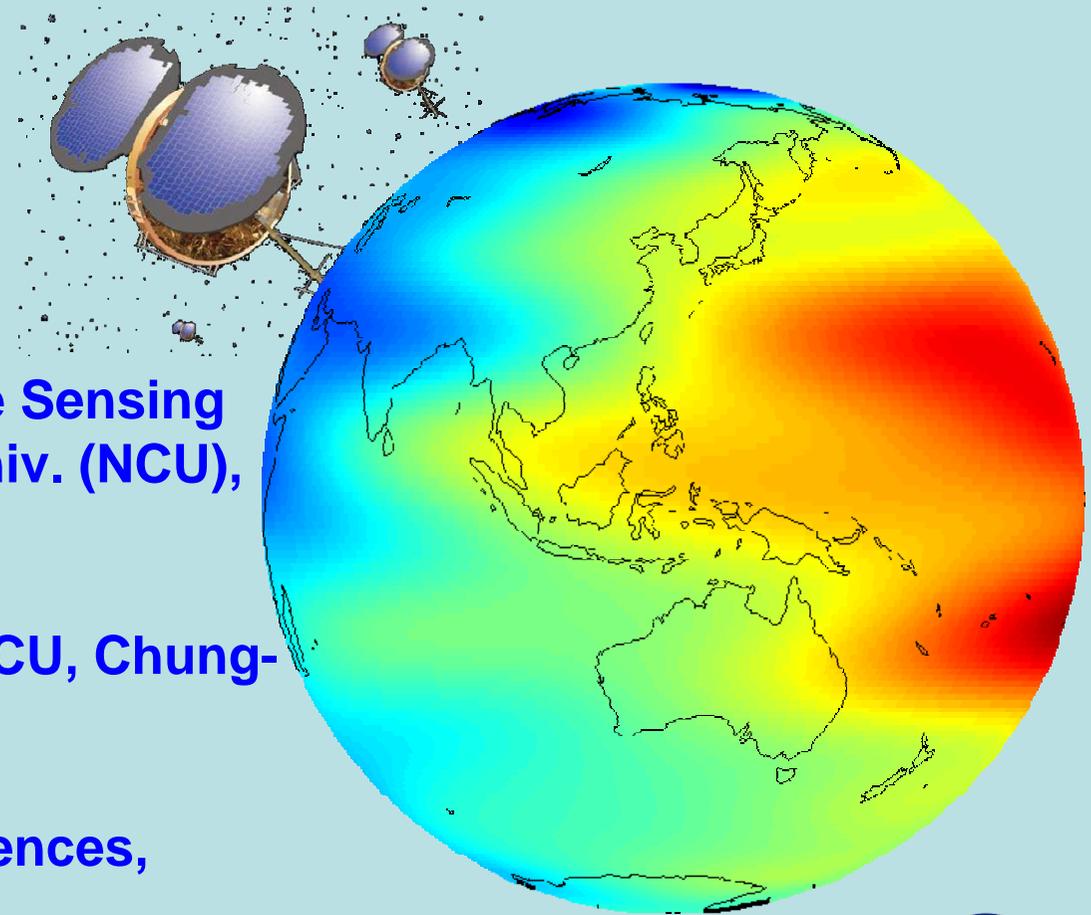


# GPS radio occultation measurements on ionospheric electron density and vTEC from low Earth orbit

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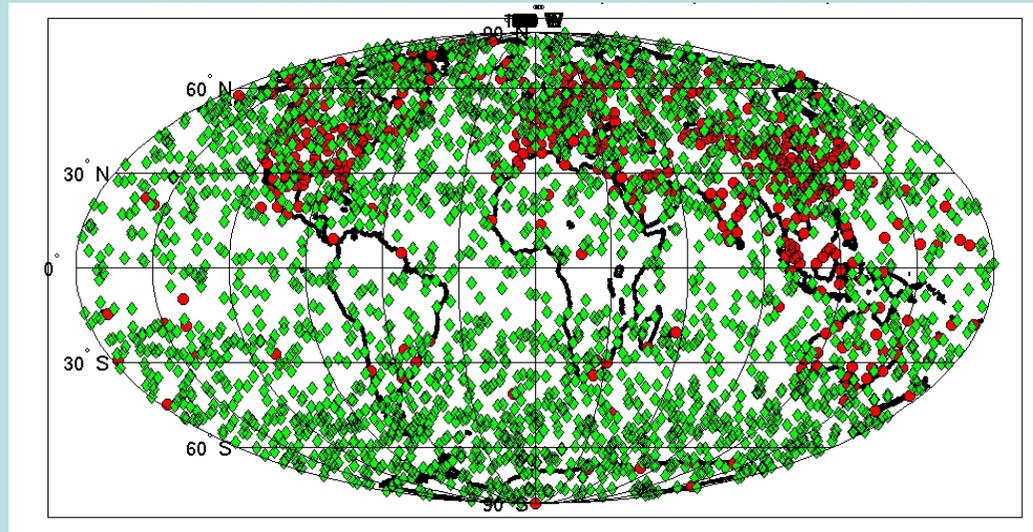
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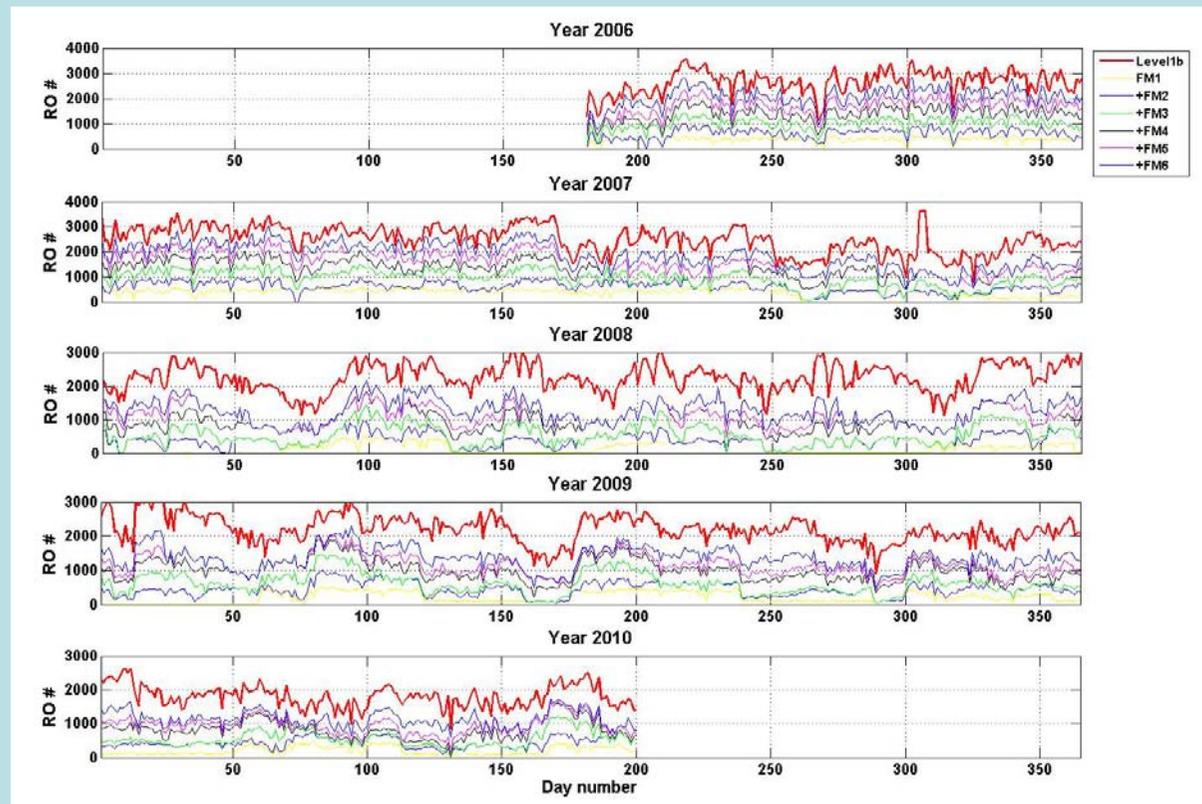
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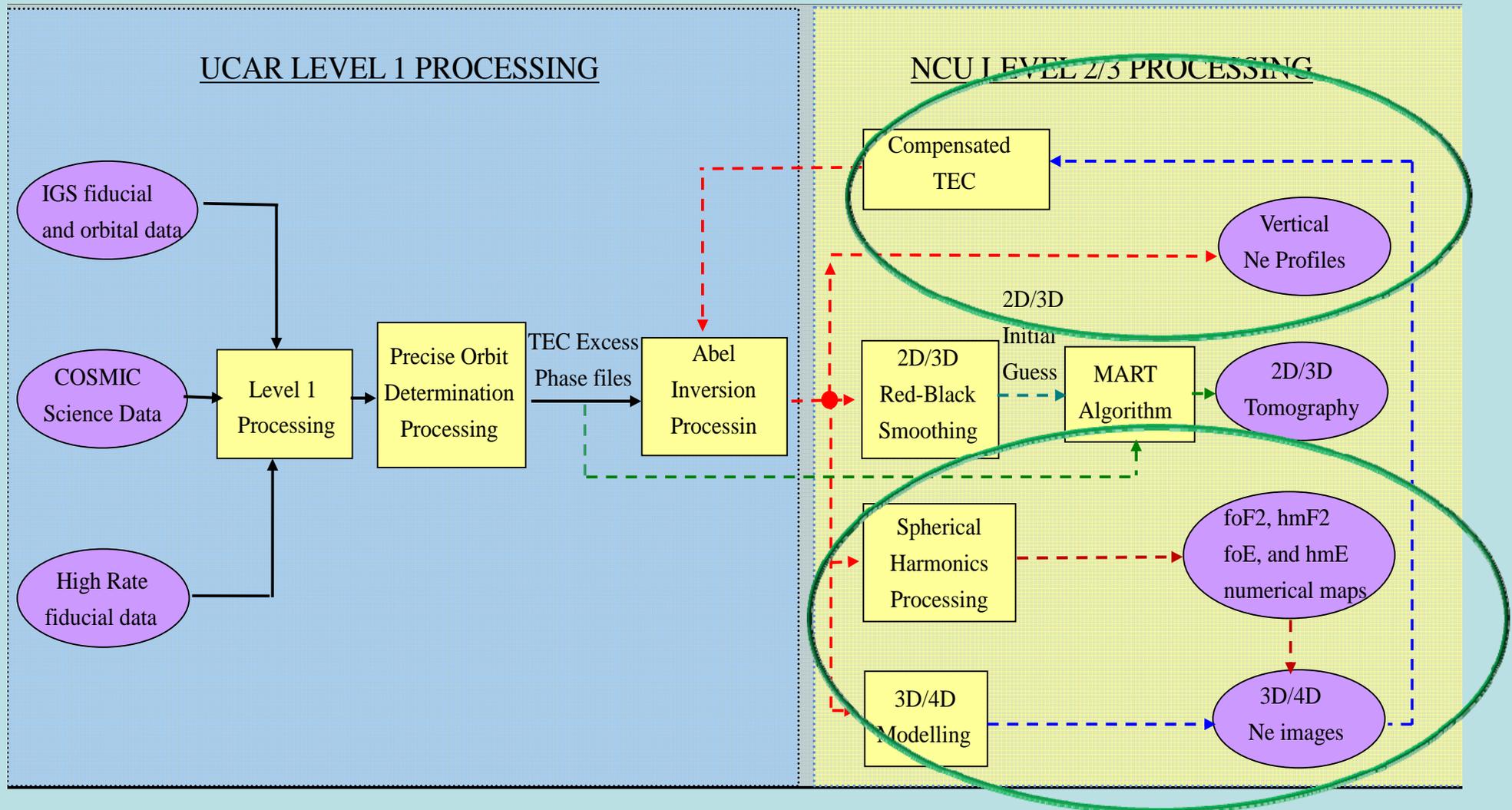


**FormoSat-3 (FS3)  
/COSMIC:  
~ 2500 radio occultation  
(RO) measurements  
worldwide daily**

**Numbers of daily RO  
observations on board  
FS3 / COSMIC and  
retrieved  $N_e$  profiles**



# Top level view of FS3/COSMIC ionospheric data processing



# GPS RO TEC observations and vertical $N_e$ retrieval

Assuming locally spherical symmetry of the ionospheric  $N_e$  and straight-line propagation for the GPS L1 and L2 signals:

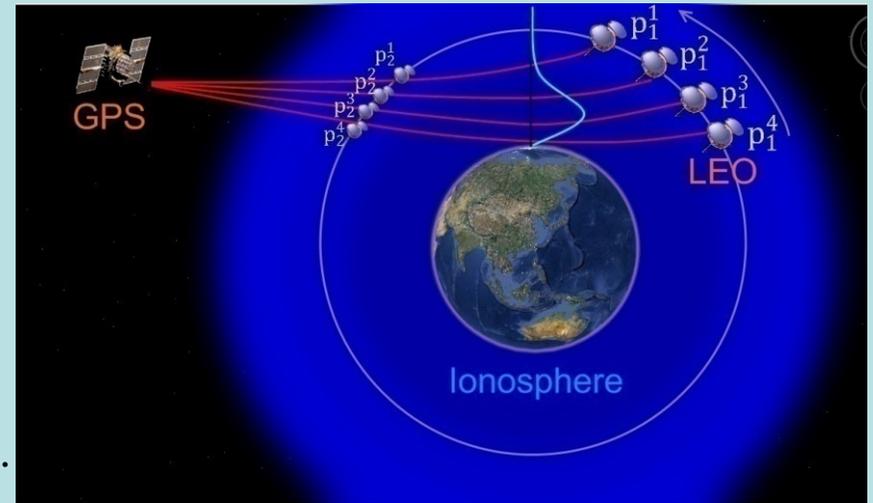
## 1. Calibrated TEC determination (TEC')

$$TEC'(r_t) = pTEC(P_1(r_t)) - pTEC(P_2(r_t)) \cong 2 \int_{r_t}^{r_{LEO}} \frac{r N_e(r)}{\sqrt{r^2 - r_t^2}} dr.$$

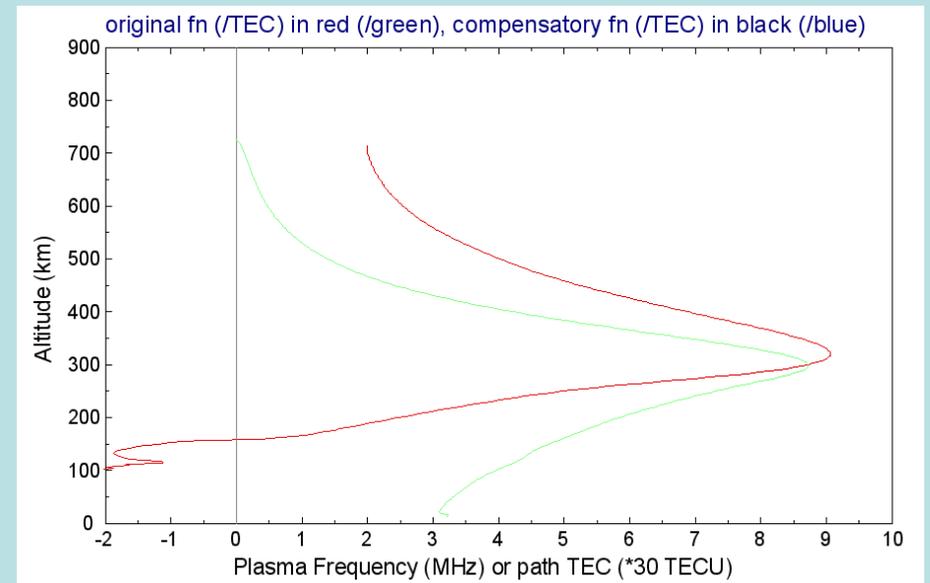
where  $pTECs$  are the path TECs from GPS to LEO, and P1 and P2 are the occulting and auxiliary positions during a GPS RO observation.

## 2. The Abel inversion through calibrated TECs

$$N_e(r_t) = -\frac{1}{\pi} \int_{r_t}^{r_{LEO}} \frac{dTEC'(r)/dr}{\sqrt{r^2 - r_t^2}} dr.$$



Example calibrated TECs (in green) and resulting  $N_e$ s (in red)



# $N_e$ retrieval errors applying the Abel inversion on TECs

- caused by the spherical symmetry assumption
- caused by the straight-line propagation (neglecting L1 & L2 ray bending) assumption
- caused by the assumption of stationary ionosphere during each occultation observation (averagely ~15 minutes)
- inadequate initial (LEO orbital)  $N_e$  and initial  $N_e$  variation below the LEO altitude
- caused by the ellipsoidal shape of the Earth
- others

# Improved Abel inversion based on compensated TECs

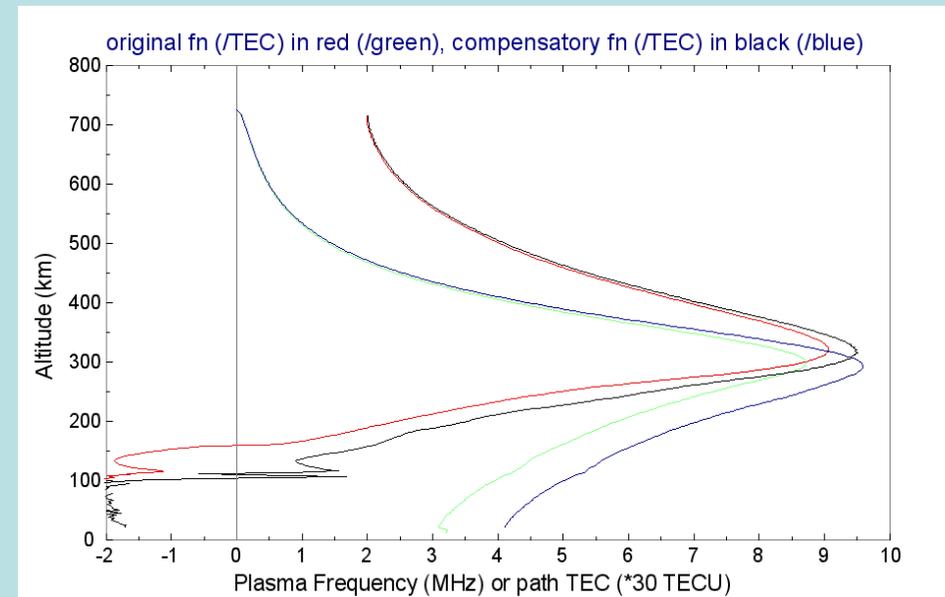
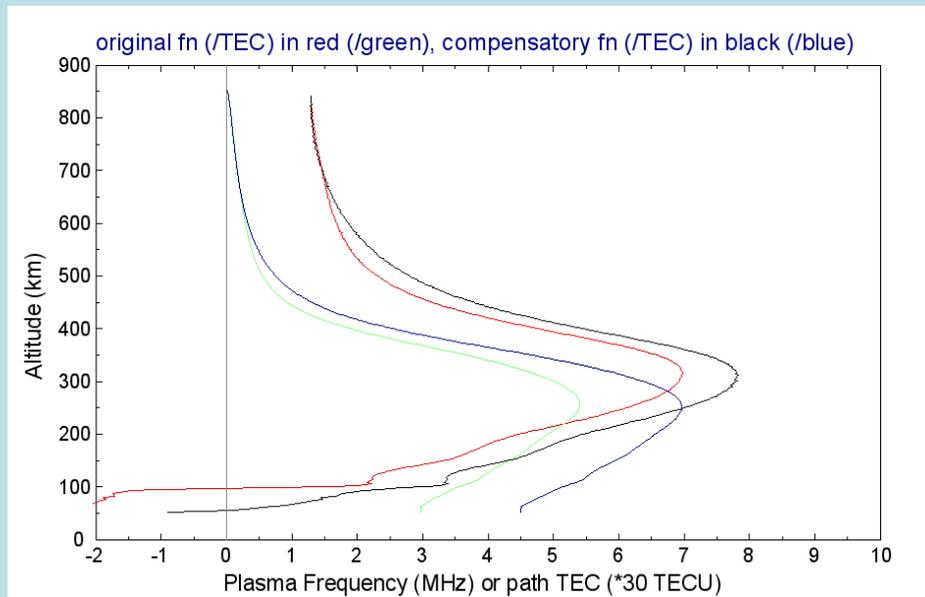
- Compensated TEC determination ( $TEC^*$ ) :

$$TEC^*(r) = TEC'(r) + \int_{P_1}^{P_2'} \left( N_e^{model}(r, \theta_t(r), \lambda_t(r)) - N_e^{model}(r, \theta, \lambda) \right) dl$$

where  $(r, \theta_t(r), \lambda_t(r))$  is the corresponding tangent-point position with the same radial distance along a ray.

- The Abel inversion through compensated TECs :

$$N_e^*(r_t) = -\frac{1}{\pi} \int_{r_i}^{r_{LEO}} \frac{dTEC^*(r) / dr}{\sqrt{r^2 - r_t^2}} dr$$



# Modelling of 3D Ionospheric Electron Density: The TaiWan Ionospheric Model (TWIM)

- Approach : vertical  $\alpha$ -Chapman layers + 2D surface spherical harmonics fitting

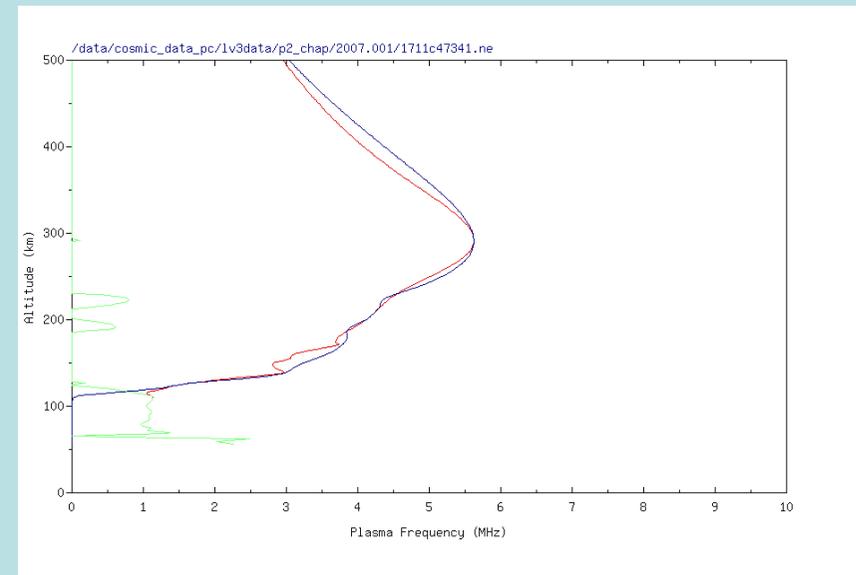
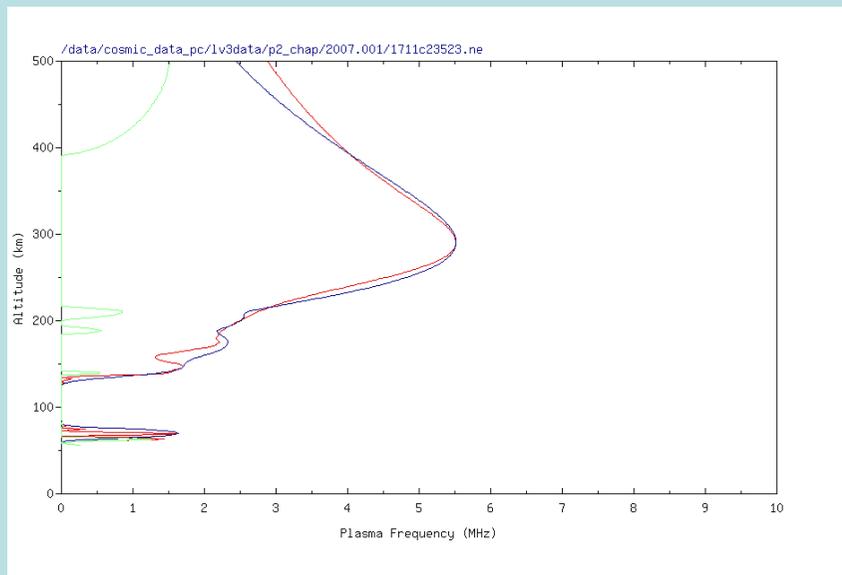
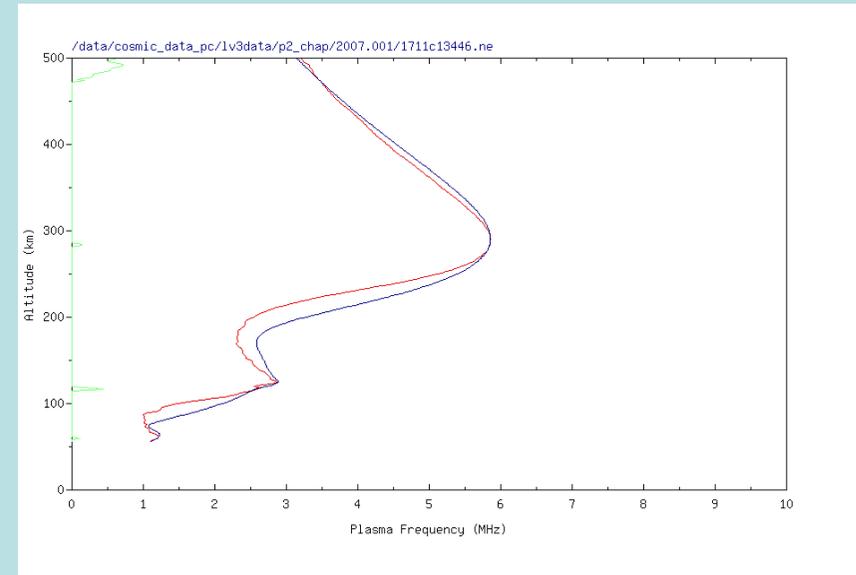
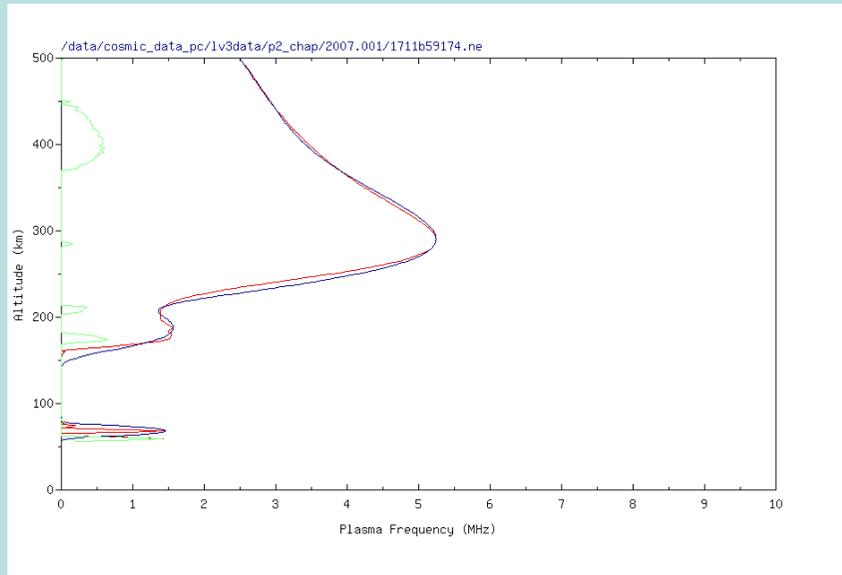
$$N_e(\theta, \lambda, h) = \sum_{i=1}^n N_{e_{\max}}(\theta, \lambda) \times e^{\frac{1}{2} \left( 1 - \frac{h-h_m(\theta, \lambda)}{H(\theta, \lambda)} - e^{-\frac{h-h_m(\theta, \lambda)}{H(\theta, \lambda)}} \right)}$$

where  $n=6$  for F2<sub>top</sub>, F2<sub>bottom</sub>, F1.5, F1, E, and D-layers, and the peak density ( $N_{e_{\max}}$ ), peak height ( $h_m$ ), and scale height ( $H$ ) are combinations of surface spherical harmonics.

- Advantage: equivalent layers for physical layers; peak height and peak density for each physical layer are explicit parameters.  
Drawback: inaccuracy of fitting profiles.

*Radio Science*, 44, doi:10.1029/2009RS004154, 2009.

# Comparisons between Chapman-layer fitting profiles (in blue) and RO $N_e$ profiles (in red)



# Numerical mapping of ionospheric parameters (e.g. $foF2$ and $hmF2$ ) by the least squares method

- The spherical surface Laplace equation is

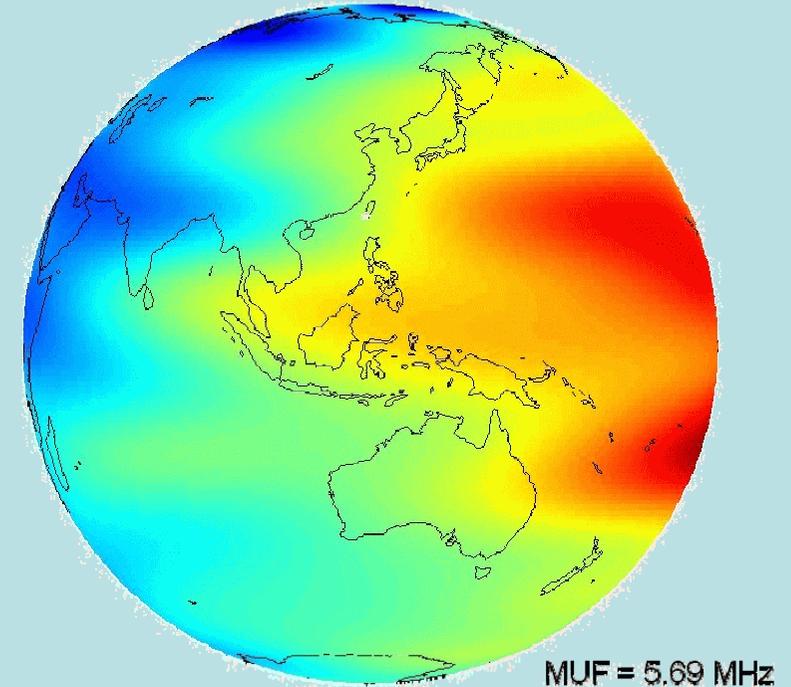
$$\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Gamma}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \Gamma}{\partial \phi^2} = 0.$$

- The resulting real and orthogonal functions are defined by

$$U_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{2\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos \theta) \cos m\phi, \text{ and}$$

$$V_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{2\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos \theta) \sin m\phi, \text{ where } n = 0, 1, 2, \dots, m = 0, 1, 2, \dots, n.$$

- Includes universal time (UT) and local time (LT) modes.



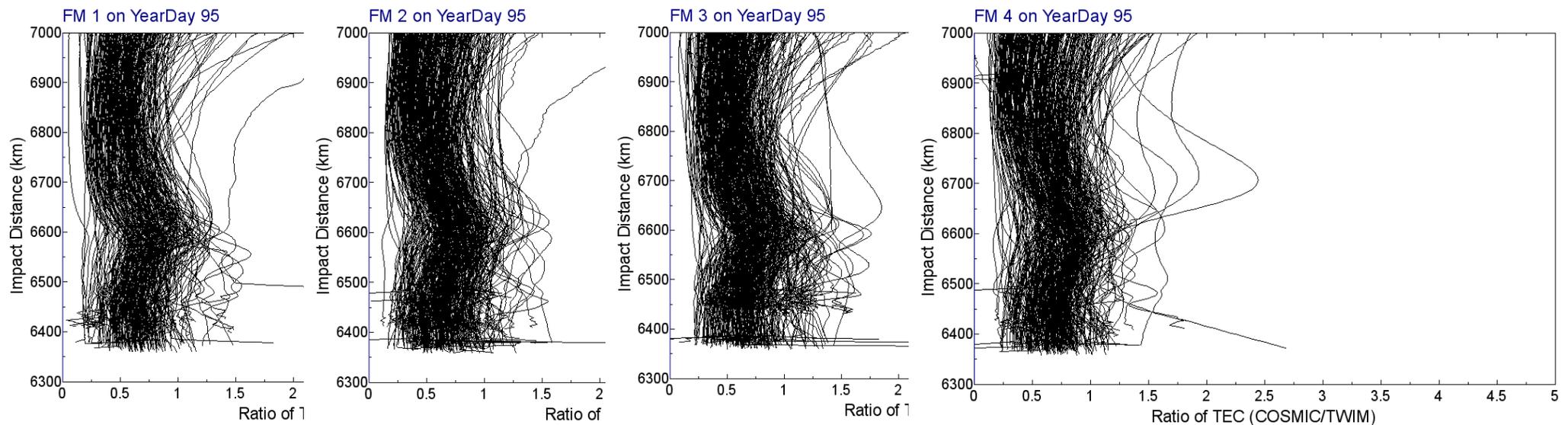
*foF2 maps in UT*

# Evaluation of TWIM limb TEC by RO measurements

- The TEC ratio (TR) profile (between the calibrated path TEC and the simulated modeling path TEC) is defined:

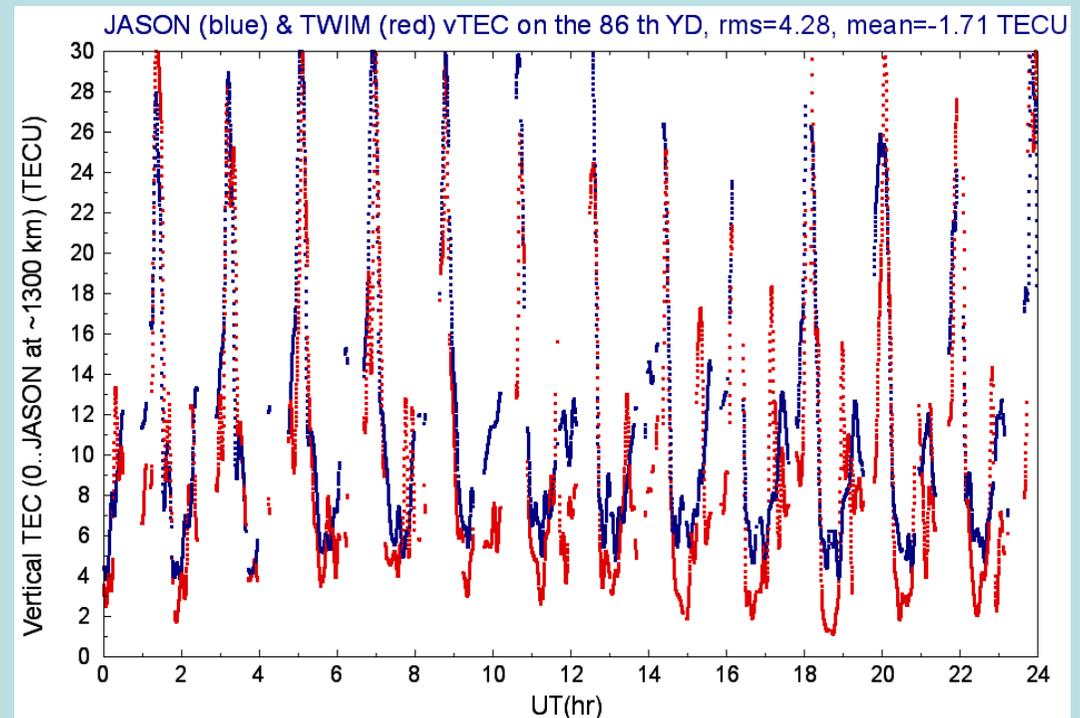
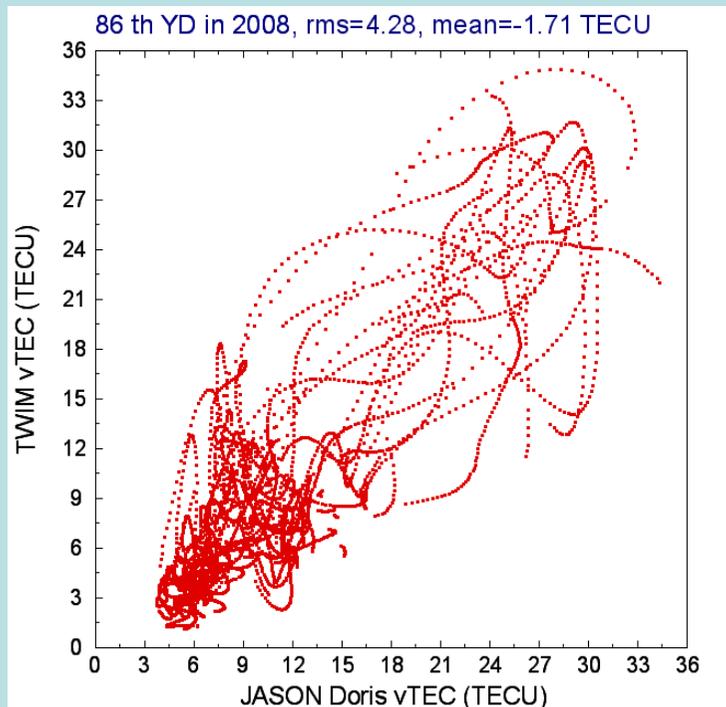
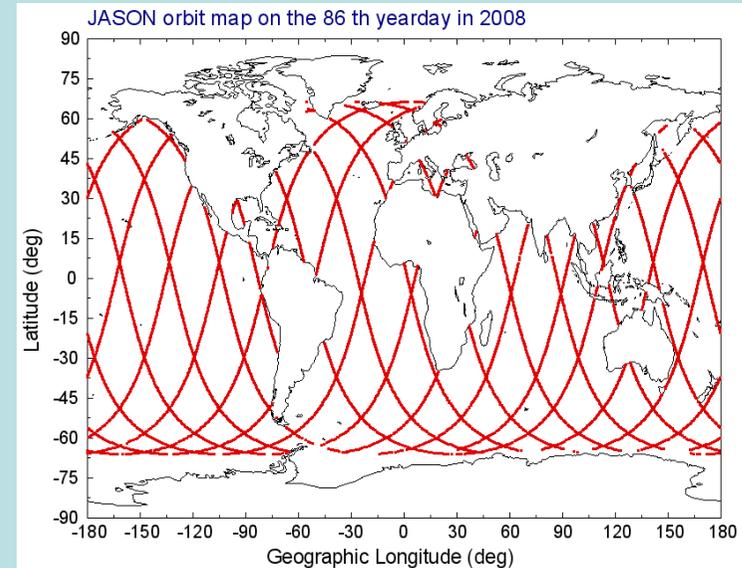
$$TR(r) = \frac{TEC'(r)}{\int_{P_1}^{P_2'} N_e^{TWIM}(r, \theta, \lambda) dl}.$$

- Examples of TR profiles on the 95<sup>th</sup> day in 2008:

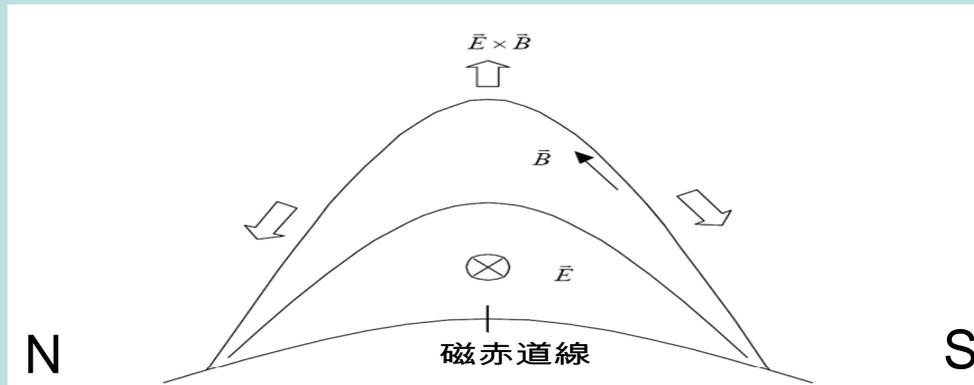


# Evaluation of TWIM vertical TEC by JASON data

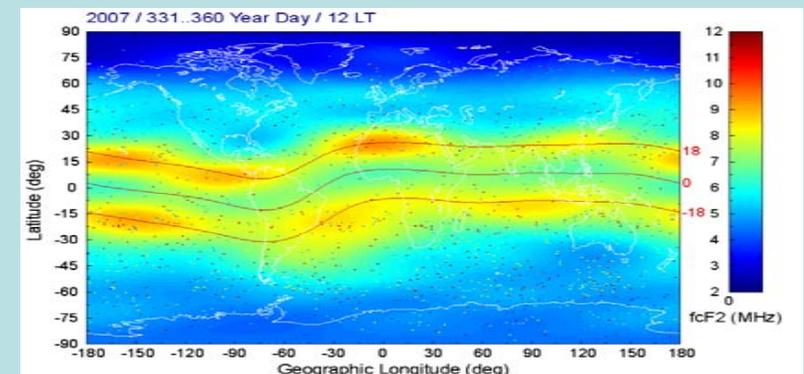
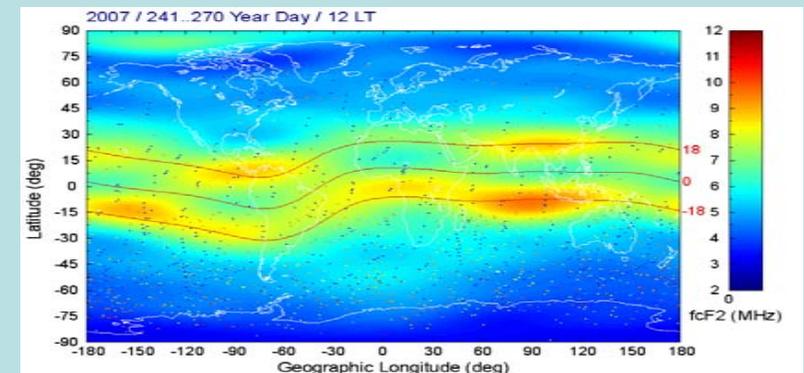
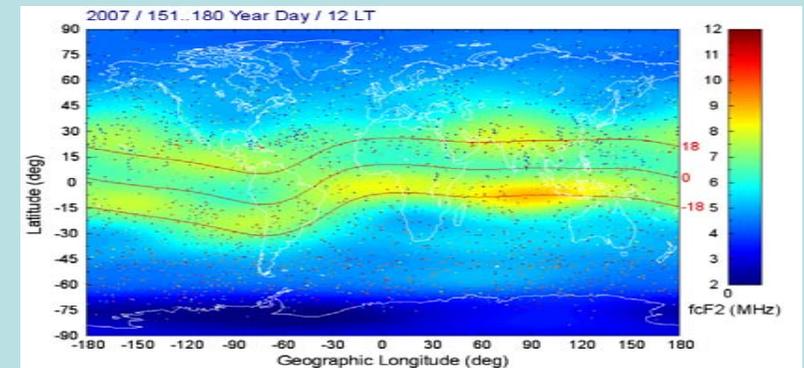
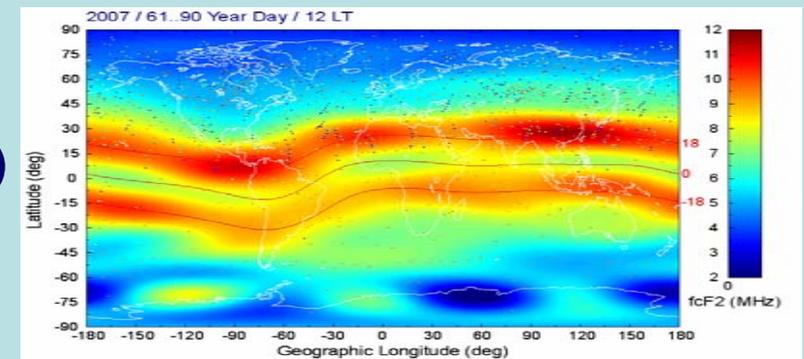
- Evaluated by JASON vTEC data (from April 2007 to Dec. 2009).
- The mean vTEC difference are  $\sim -2$  TECU. The *rms* vTEC differences are  $\sim 4.5$  TECU.
- Examples on the 86<sup>th</sup> day in 2008:



# March, June, September, and December noontime $foF2$ maps (from top to bottom)

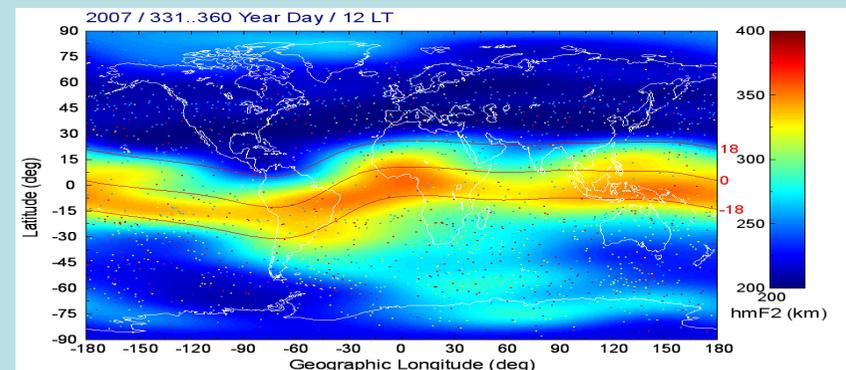
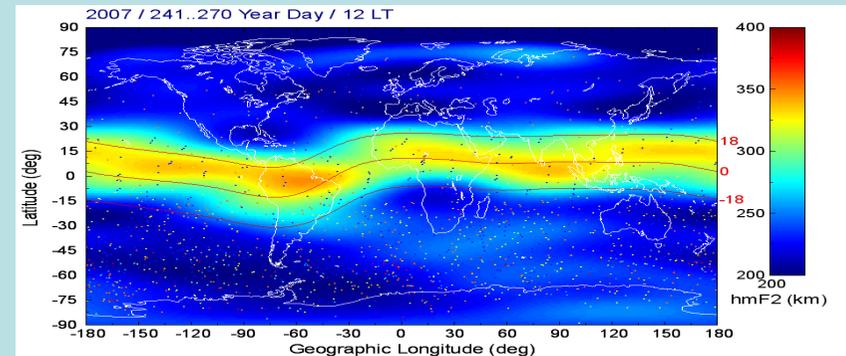
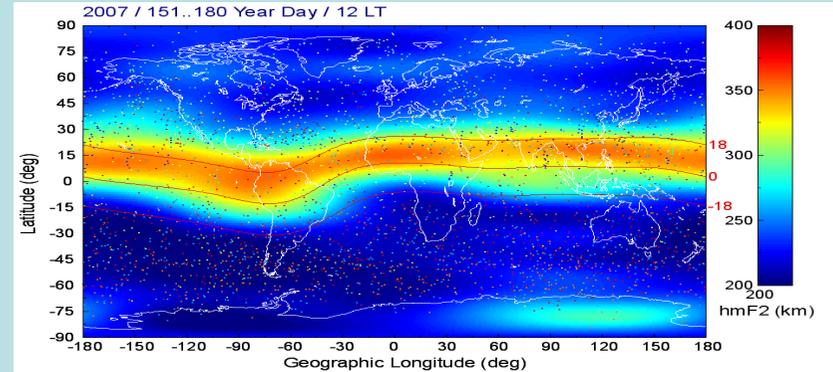
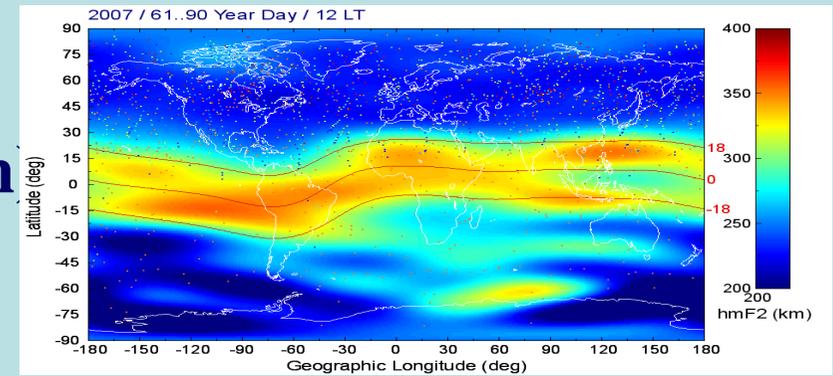


1. The June noontime  $foF2$  map shows that the north-pole area has higher values ( $\sim 5$  MHz) than the south-pole area ( $\sim 2$  MHz), which is in continuous night during the winter.
2. All the noontime  $foF2$  maps show one equatorial  $N_e$  trough along the dip equator and two  $N_e$  crest traces at  $\pm \sim 18^\circ$  magnetic dip latitudes for equatorial anomaly (EA).
3. For seasonal variations of EA at noon, the  $foF2$  values along the two crests are highest at the March equinox and lowest at the June solstice.



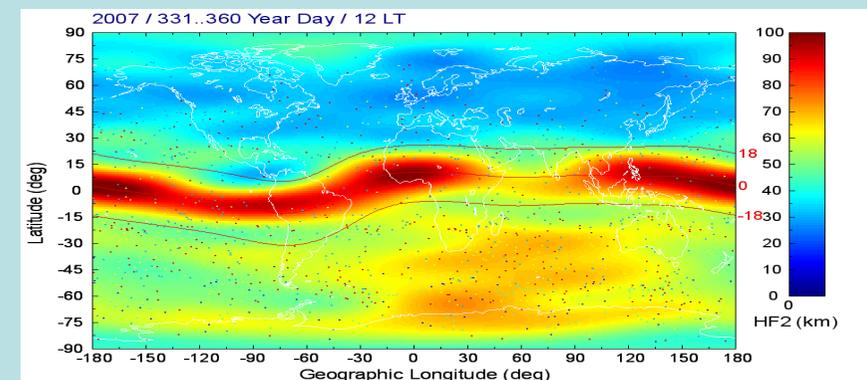
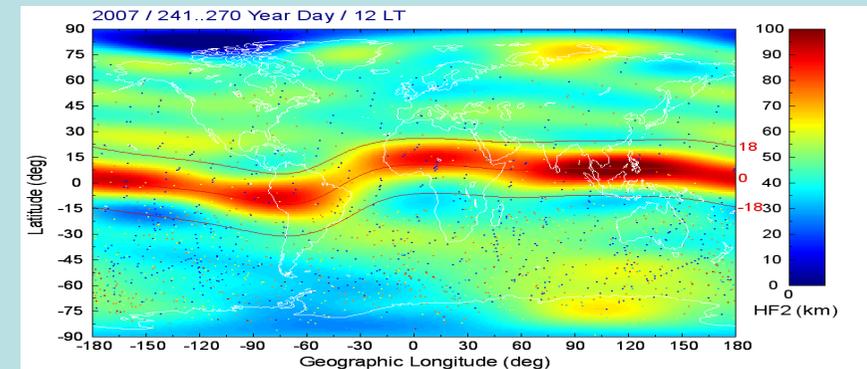
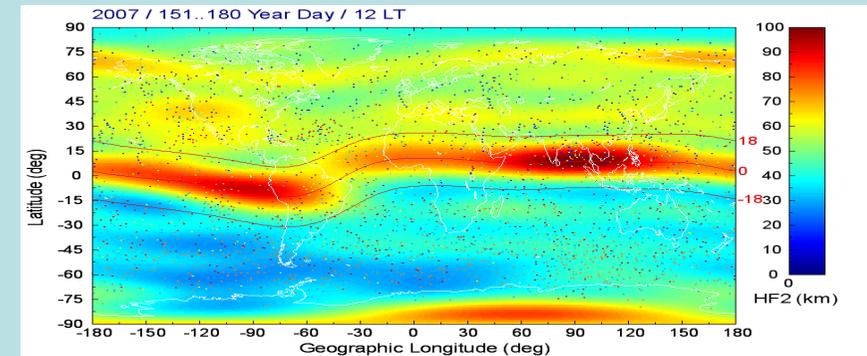
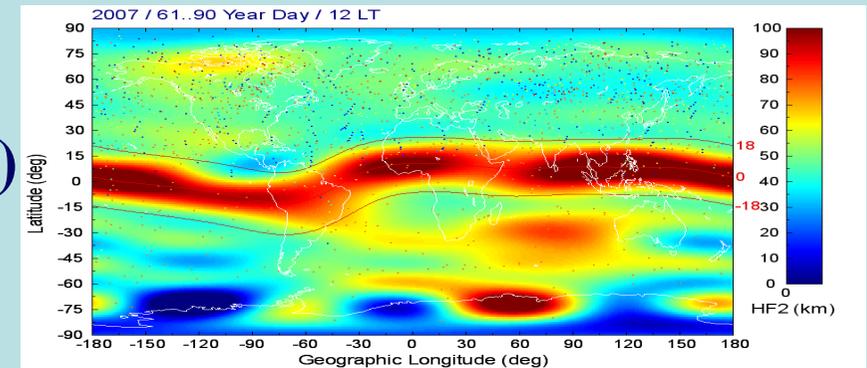
# March, June, September, and December noontime $hmF2$ maps (from top to bottom)

1. From the June noontime  $hmF2$  map the mid- and high-latitude  $hmF2$ s in the northern hemisphere generally have relatively higher values (by 30-40 km) than the southern hemisphere.
2. Summer-to-winter wind effects can transport plasma up the field line and produce higher  $hmF2$ s in the northern hemisphere and low dip latitude regions, as shown in the June  $hmF2$  maps; there are lower  $hmF2$ s in the southern hemisphere and low dip latitude regions vice versa.



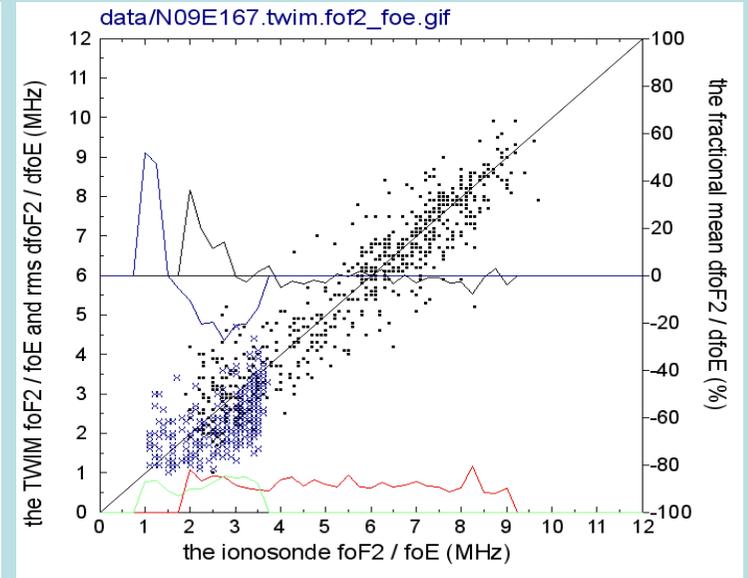
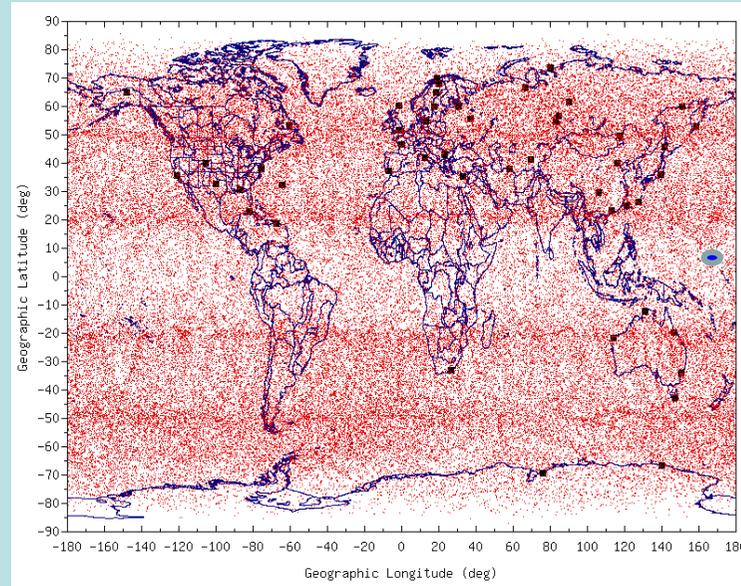
# March, June, September, and December noontime *HF2* maps (from top to bottom)

1. The June noontime *HF2* map shows that the mid- and high-latitude *HF2*s in the northern hemisphere generally have relatively thicker values (by 10-20 km) than the southern hemisphere.
2. Summer-to-winter wind effects can also transport plasma up the field line and produce thicker *HF2*s in the northern hemisphere and low dip latitude regions, as shown in the June *HF2* maps; there are lower *HF2*s in the southern hemisphere and low dip latitude regions vice versa.
3. The noontime *HF2* values display peaks along the dip equator and are largest in March and lowest in June.
4. For seasonal variations of EA at noon, the *HF2* values along the dip equator are highest at the March equinox and lowest at the June solstice.

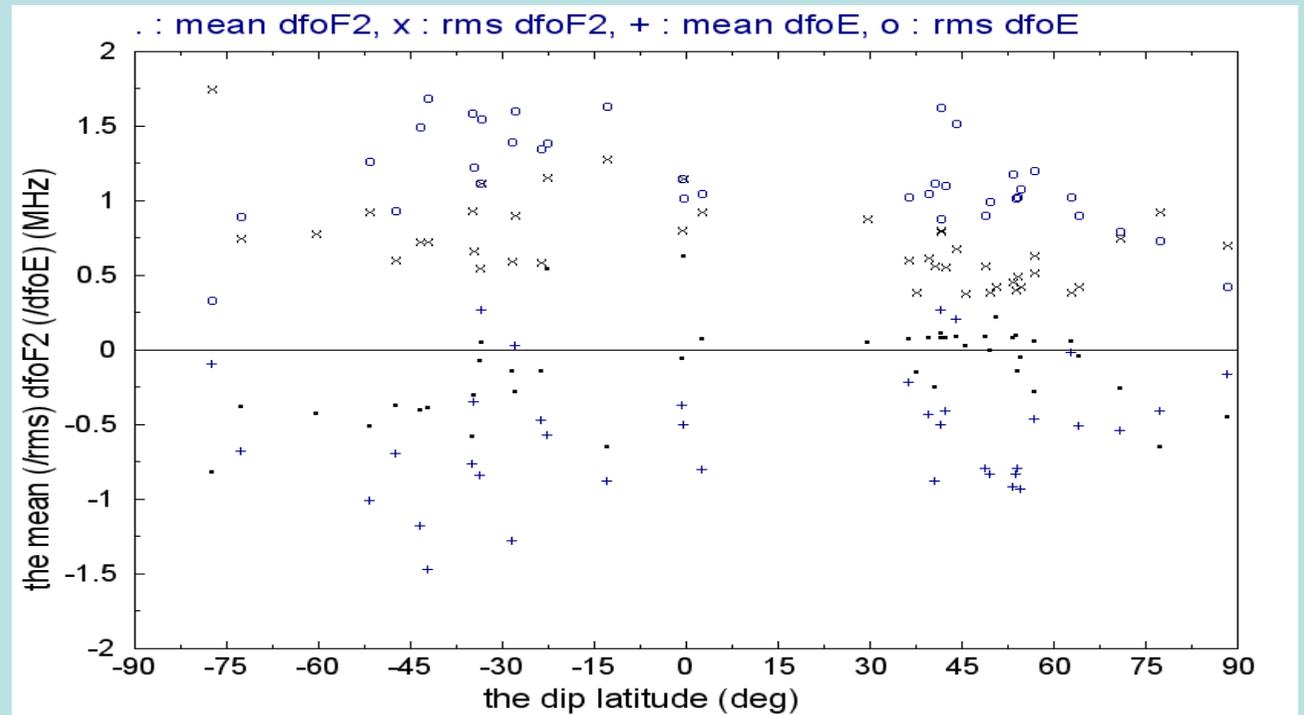


# Validation of compensated TEC retrieved $foF2$ and $foE$ by the ionosonde data

- Evaluated by ionosonde data (49 stations) from days 81~180 in 2008.
- ~4000 matches out of ~150,000 RO observations.



- mean (rms)  $foF2$  difference = -0.12 (0.76) MHz
- mean (rms)  $foE$  difference = -0.37 (0.94) MHz
- The mean  $dfoF2$ s are  $\leq 0.5$  MHz averagely and  $\leq 0.25$  MHz from  $0^\circ$  to  $60^\circ$  dip-latitude region.
- The retrieved  $foE$  values are underestimated and typically worse at the southern hemisphere.



## Final remarks

- FormoSat-3/COSMIC has become a promising program for monitoring the large-scale global ionosphere.
- A 3D approach using vertical Chapman layers and surface spherical harmonics for modeling variations of ionospheric Ne have been proposed and implemented to be the TaiWan Ionospheric Model (TWIM).
- The standard Abel inversion error because of potential horizontal gradient and inhomogeneities of the ionosphere could be reduced by compensating path TECs based on a qualified ionospheric model, e.g, TWIM.

## Future works

- Including ionosonde and ISR measurements to improve the TWIM results.