RECONSTRUCTION OF INTERNAL GRAVITY WAVE PARAMETERS FROM RADIO OCCULTATION RETRIEVALS OF VERTICAL TEMPERATURE PROFILES IN THE EARTH ATMOSPHERE

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ABSTRACT The new method for the reconstruction of internal gravity wave (IGW) parameters from a single vertical temperature profile measurement in the Earth atmosphere has been developed. This method does not require any additional information not contained in the profile and may be used for the analysis of profiles measured by various techniques. The criterion for the IGW identification has been formulated and argued. In the case when this criterion is satisfied, then analyzed temperature fluctuations can be considered as wave-induced. The method is based on the analysis of relative amplitude thresholds of the temperature wave field and on the linear IGW saturation theory in which amplitude thresholds are restricted by dynamical (shear) instability processes in the atmosphere. When the amplitude of an internal gravity wave reaches the shear instability limit, energy is assumed to be dissipated in such a way that the amplitude is maintained at the instability limit as the wave propagates upwards.

In order to approbate the method we have used in situ data of simultaneous balloon high-resolution measurements of the temperature and wind velocity in the Earth stratosphere (France) where a long-period inertia-gravity wave has been detected. Using the temperature data only, we have reconstructed all the measured wave parameters with uncertainties not larger than 30%. An application of the method to the radio occultation data has given the possibility to identify the IGW in the Earth stratosphere and to determine the magnitudes of key wave parameters such as the intrinsic frequency, amplitudes of vertical and horizontal perturbations of the wind velocity, vertical and horizontal wavelengths, intrinsic vertical and horizontal phase (or group) speeds, kinetic and potential energy, vertical fluxes of the wave energy and horizontal momentum. The obtained results of internal wave studies in the Earth stratosphere deduced from the COSMIC and CHAMP GPS occultation temperature profiles have been presented and discussed.

The aim of this work is the:

1) formulation and argumentation of the identification criterion for a low-frequency wave which saturates via shear instability;

2) development of an analysis technique for determination the intrinsic frequency and other wave parameters from a single vertical temperature or density profile;

3) verification of the developed technique on the base of simultaneous *in situ* balloon measurements of the temperature and wind velocity in the Earth' stratosphere where the saturated internal wave has been detected;

4) practical application of the proposed technique on radio occultation retrievals of temperature profiles in the Earth' atmosphere.

Transmitter GPS

Ionosphere

Neutral atmosphere

Receiver
Occulting LEO

Earth
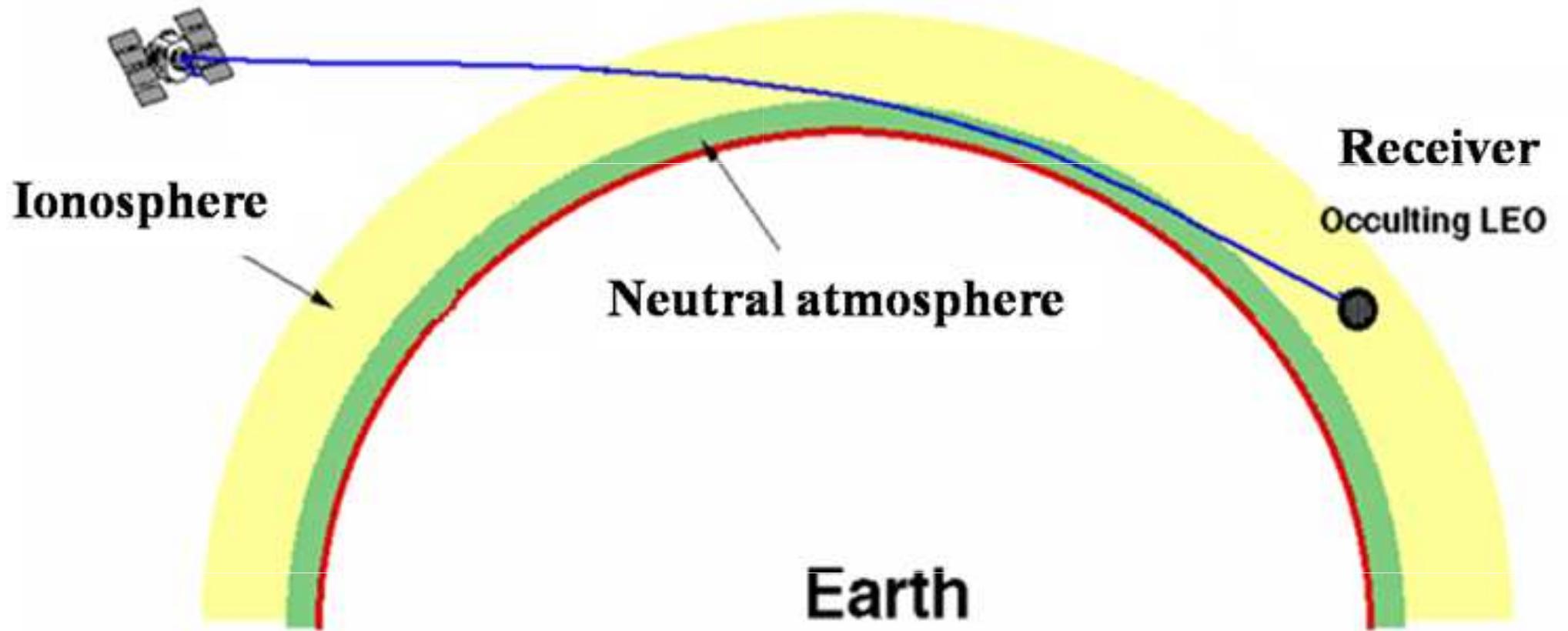
The occultation geometry involving the GPS transmitter and receiver

THEORETICAL RELATIONSHIPS [*Gubenko et al.*, 2008]

$N^2 >> \omega^2 > f^2$ *Approximation of medium and low frequency waves*

$$\left| C_{ph}^{in} \right|^2 = \left| c_h - \bar{u} \right|^2 = \frac{\omega^2}{k^2 + l^2} = \frac{N^2}{m^2} \cdot \frac{1}{1 - f^2 / \omega^2}$$

Dispersion relation for IGW

Polarization equations for IGW

Relative amplitude threshold theory

$$\left| u' \right| = \frac{g}{N} \cdot \frac{\left| T' \right|}{\bar{T}} \cdot \left(1 - f^2 / \omega^2 \right)^{-1/2} = \frac{g}{N} \cdot \frac{\left| \rho' \right|}{\bar{\rho}} \cdot \left(1 - f^2 / \omega^2 \right)^{-1/2}$$

$$a = \frac{\left| u' \right|}{\left| c_h - \bar{u} \right|} = \frac{2 \cdot \left(1 - f^2 / \omega^2 \right)^{1/2}}{1 + \left(1 - f^2 / \omega^2 \right)^{1/2}}$$

Criterion for IGW identification theory

experiment

$$1 > a = a_e > 0$$

$$a_e = \frac{g \left| m \right|}{N^2} \cdot \frac{\left| T' \right|}{\bar{T}} = \frac{g \left| m \right|}{N^2} \cdot \frac{\left| \rho' \right|}{\bar{\rho}} = \frac{2 \left(1 - f^2 / \omega^2 \right)^{1/2}}{1 + \left(1 - f^2 / \omega^2 \right)^{1/2}}$$

Brunt-Vaisala frequency squared

Coriolis parameter $f = 2\Omega \cdot \sin\theta$

$$N^2 = \frac{g}{T} \cdot \left(\frac{\partial T}{\partial z} + \frac{g}{c_p} \right)$$

Horizontal wave number squared $k_h^2 = k^2 + l^2$

Determination of IGW parameters on the basis of a vertical temperature profile in a planet atmosphere

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┌─────────────────┐    ┌──────────────────────┐    ┌──────────────────────┐
│ Raw temperature │───▶│ Determination of the │───▶│ Estimation of the    │
│ versus altitude │    │ background           │    │ dominant             │
│ profile         │    │ temperature profile  │    │ vertical wave number │
│                 │    │ and the relative     │    │                      │
│                 │    │ temperature          │    │                      │
│                 │    │ fluctuations         │    │                      │
│                 │    │ amplitude            │    │                      │
└─────────────────┘    └──────────────────────┘    └──────────────────────┘
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- Raw temperature versus altitude profile
- Determination of the background temperature profile and the relative temperature fluctuations amplitude
- Estimation of the dominant vertical wave number
- Derivation of the vertical profile of Brunt-Vaisala frequency
- Evaluation of the relative amplitude threshold a_e
- Are the IGW identification criterion for the amplitude threshold satisfied?
- Determination of the IGW intrinsic frequency
- Computation of the Coriolis parameter f (inertial frequency)
- YES
- NO
- Determination of other IGW parameters

DETERMINED IGW PARAMETERS [*Gubenko et al., 2008*]

$$\omega = \frac{f}{2} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}}$$

Intrinsic frequency

Horizontal wave number

$$|k_h| = \frac{\omega}{|c_h - \bar{u}|} = \frac{|m|}{2} \cdot \frac{f}{N} \cdot \frac{a_e}{(1 - a_e)^{1/2}} = \frac{\pi \cdot f}{\lambda_z N} \cdot \frac{a_e}{(1 - a_e)^{1/2}}$$

Intrinsic horizontal phase speed

$$\left| C_{ph}^{in} \right| = |c_h - \bar{u}| = \frac{N}{|m|} \cdot \frac{2 - a_e}{a_e} = \frac{\lambda_z N}{2\pi} \cdot \frac{2 - a_e}{a_e}$$

Horizontal wavelength

$$\lambda_h = \frac{2\pi}{|k_h|} = 4\pi \frac{N}{f} \cdot \frac{(1 - a_e)^{1/2}}{a_e \cdot |m|} = 2 \frac{\lambda_z N}{f} \cdot \frac{(1 - a_e)^{1/2}}{a_e}$$

$$\left| C_{pz}^{in} \right| = \frac{\omega}{|m|} = \frac{f}{2|m|} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}} = \frac{\lambda_z f}{4\pi} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}}$$

Intrinsic vertical phase speed

$$|w'| = \frac{|k_h|}{|m|} \cdot |u'| = \frac{f \cdot a_e}{2|m|} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}} = \frac{\lambda_z f \cdot a_e}{4\pi} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}}$$

Amplitude of vertical perturbations of wind speed

Amplitudes of horizontal perturbations of wind speed

$$|u'| = a_e |c_h - \bar{u}| = \frac{N}{|m|} \cdot (2 - a_e) = \frac{\lambda_z N}{2\pi} \cdot (2 - a_e)$$

$$|v'| = \frac{f}{\omega} \cdot |u'| = 2 \frac{N}{|m|} (1 - a_e)^{1/2} = \frac{\lambda_z N}{\pi} (1 - a_e)^{1/2}$$

in direction of horizontal component of wave vector

in direction normal to horizontal component of wave vector

DETERMINED ENERGY IGW PARAMETERS

$$\mathrm{tg}\varphi' = \frac{|m|}{|k_h|} = \frac{\lambda_h}{\lambda_z}$$ *Tan of the angle between wave vector and horizontal plane*

Wave kinetic energy per mass unit $\quad E_k = \frac{1}{2}\left[\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right] = E \cdot \frac{1 + (f/\omega)^2 \sin^2\varphi'}{2}$

Wave potential energy per mass unit

$$E_p = \frac{1}{2}\frac{g^2}{N^2}\overline{\left(\frac{T'}{\overline{T}}\right)^2} = \frac{1}{4}\frac{g^2}{N^2}\left|\frac{T'}{\overline{T}}\right|^2 = \frac{1}{2}\frac{g^2}{N^2}\overline{\left(\frac{\rho'}{\overline{\rho}}\right)^2} = \frac{1}{4}\frac{g^2}{N^2}\left|\frac{\rho'}{\overline{\rho}}\right|^2 = E \cdot \frac{1 - (f/\omega)^2 \sin^2\varphi'}{2}$$

$$E = E_k + E_p = \frac{1}{2}|w'|^2\left(1 + \mathrm{tg}^2\varphi'\right) = \frac{1}{2}\left(|w'|^2 + |u'|^2\right)$$ *Total wave energy per mass unit*

Ratio of kinetic to potential energy $\quad p = \dfrac{E_k}{E_p} = \dfrac{\omega^2 + f^2 \sin^2\varphi'}{\omega^2 - f^2 \sin^2\varphi'} = 1 + 2\dfrac{f^2}{N^2}\mathrm{tg}^2\varphi'$

$$\left|C_{gh}^{in}\right| = \left|\frac{\partial\omega}{\partial k_h}\right| = \left|C_{ph}^{in}\right| \cdot \left(1 - \frac{f^2}{\omega^2}\right) = \frac{N}{|m|} \cdot \sqrt{1 - \frac{f^2}{\omega^2}}$$ *Intrinsic horizontal group speed*

Intrinsic vertical group speed $\quad \left|C_{gz}^{in}\right| = \left|\dfrac{\partial\omega}{\partial m}\right| = \left|C_{pz}^{in}\right| \cdot \left(1 - \dfrac{f^2}{\omega^2}\right) = \dfrac{|k_h|N}{m^2} \cdot \sqrt{1 - \dfrac{f^2}{\omega^2}}$

FLUXES OF WAVE ENERGY AND HORIZONTAL MOMENTUM DUE TO IGW

$$\left|\frac{C_{gz}^{in}}{C_{gh}^{in}}\right| = \left|\frac{C_{pz}^{in}}{C_{ph}^{in}}\right| = \left|\frac{k_h}{m}\right| = \frac{\lambda_z}{\lambda_h} = \frac{\omega}{N} \cdot \sqrt{1 - \frac{f^2}{\omega^2}}$$

Ratio of vertical to horizontal group (phase) speed

$$\left|F_z\right| = \left|C_{gz}^{in}\right| \cdot E$$

Vertical flux of wave energy

$$\left|F_h\right| = \left|C_{gh}^{in}\right| \cdot E$$

Horizontal flux of wave energy

$$\left|F_{ph}\right| = \left|\overline{u' \cdot w'}\right| = \frac{\left|u'\right| \cdot \left|w'\right|}{2} = 2\left|\frac{k_h}{m}\right| E_p$$

Total vertical flux of horizontal momentum due to IGW

For the experimental verification of the analysis technique proposed, we used the results of the simultaneous temperature and wind velocity measurements obtained in a high-resolution balloon experiment [*Cot and Barat*, 1986], where a nearly monochromatic and long-period wave propagating upward in the stratosphere was identified. From all the data, *Cot and Barat* [1986] have estimated the mean characteristics of the wave, such as the vertical and horizontal wavelengths, ratio f/ω, vertical and horizontal wave-induced velocity amplitudes, intrinsic vertical and horizontal phase speeds, temperature perturbation amplitude. By using the temperature data only, we reconstructed the ratio f/ω and other wave parameters. In Table 1 we give observed wave parameters [*Cot and Barat*, 1986], our reconstruction results, and relative deviations of observed and reconstructed wave parameters. An comparison between observed and reconstructed wave parameters shows good agreement. It is seen from the values given in Table 1 that the relative deviations of the reconstructed parameters are not larger than 31%. The best agreement is found for such parameters as f/ω, ω, W_Φ, $|w'|$ which are reproduced with relative deviations 10%. Thus we have shown that reasonable estimates of wave characteristics can be obtained from a single temperature profile in the case when the wave is positively identified.

Table 1. Summary of observed wave parameters [*Cot and Barat,* 1986] and those reconstructed from temperature data. Relative deviations between observed and reconstructed wave parameters are indicated

λ_z, km	$|T'|$,K	$\bar{T}$, K	N, rad/s	a_e	f/ω	$\omega\cdot10^4$, rad/s	$|c-\bar{u}|$, m/s	$|u'|$, m/s	λ_x, km	W_Φ, m/s	$|w'|$, m/s
Parameters observed by *Cot and Barat* [1986]											
1.0	1.0	231.5	0.02	–	0.8	1.25	5.2	3.3	260	0.020	0.013
Basic parameters for reconstruction				Parameters reconstructed from temperature data							
1.0	1.0	231.5	0.02	0.67	0.86	1.16	6.3	4.2	341	0.018	0.012
				Relative deviations of observed and reconstructed wave parameters, %							
				–	7.5	7.2	21.2	27.3	31.2	10.0	7.7

ERRORS OF THE RELATIVE AMPLITUDE THRESHOLD DETERMINATION [*Gubenko et al., 2008*]

$$a_e = \frac{g\,|m|}{N^2}\cdot\frac{|T\,'|}{\overline{T}} = \frac{2\pi\cdot g}{\lambda_z N^2}\cdot\frac{|T\,'|}{\overline{T}} = \frac{2\left(1-f^2\big/\omega^2\right)^{1/2}}{1+\left(1-f^2\big/\omega^2\right)^{1/2}}$$

Relative amplitude threshold

Relative uncertainty for the amplitude threshold determination

$$X = \frac{\delta a_e}{a_e} \approx \left[\left(\frac{\delta|T\,'|}{|T\,'|}\right)^2 + \left(\frac{\delta\lambda_z}{\lambda_z}\right)^2 + \left(\frac{\delta N^2}{N^2}\right)^2\right]^{1/2} \approx \left[\frac{\lambda_z}{L}\left(\frac{\delta T}{|T\,'|}\right)^2 + \frac{\lambda_z}{L}\left(\frac{\delta h}{\lambda_z}\right)^2 + 2\pi^2\left(\frac{|T\,'|}{\lambda_z}\right)^2\cdot\left(\frac{\partial\overline{T}}{\partial z}+\frac{g}{c_p}\right)^{-2}\right]^{1/2}$$

$$1 > a = a_e > 0$$ *Theoretical limitation for the amplitude threshold*

Restrictions for the determined amplitude threshold in the real experiment

$$1 > a_e + \delta a_e > a_e > a_e - \delta a_e > 0 \quad\Leftrightarrow\quad 1 > (1+X)\,a_e > a_e > (1-X)\,a_e > 0$$

Minimal amplitude of temperature fluctuations applicable to identify IGW

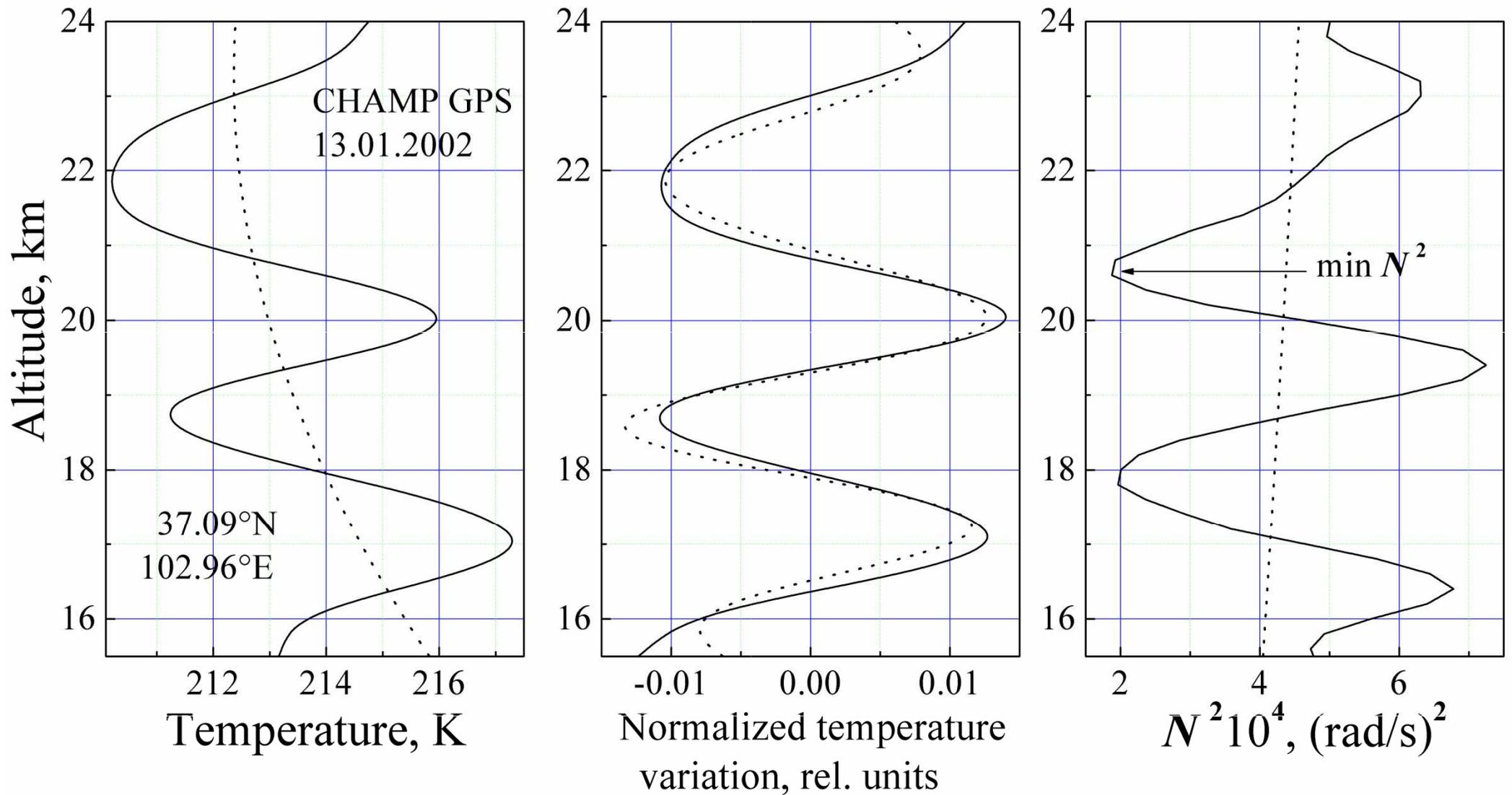$$min\,|T\,'| = \delta T\,/\,[L/\lambda_z - (\delta h\,/\,\lambda_z)^2]^{1/2}$$

Figure 1. Example of IGW observations in the middle latitudes of the Earth's atmosphere from the temperature $T(h)$ profile measured by CHAMP GPS radio occultation on January 13, 2002. The coordinates of sounded region, date of observation are indicated.

In successive panels of Figure 1, from left to right, are plotted the altitude profiles of temperature, normalized temperature variation, and Brunt-Vaisala frequency squared N^2. The solid line (left panel) is the raw temperature data, and the dotted line is the background temperature profile determined by a least-squares cubic polynomial curve fit. Based on these raw and background profiles, we computed the corresponding original (solid line) and mean (dotted line) profiles of N^2 (right panel). We used a low-pass digital filter with a cutoff at 2.0 km to calculate the smoothed profile (dotted line, middle panel) of normalized temperature variation from raw data (solid line, middle panel). The strong wave-like oscillations of T and N^2 have the dominant vertical wavelength about 3.2 km, and these are probably caused by an internal gravity wave (see IGW parameters in Table 2).
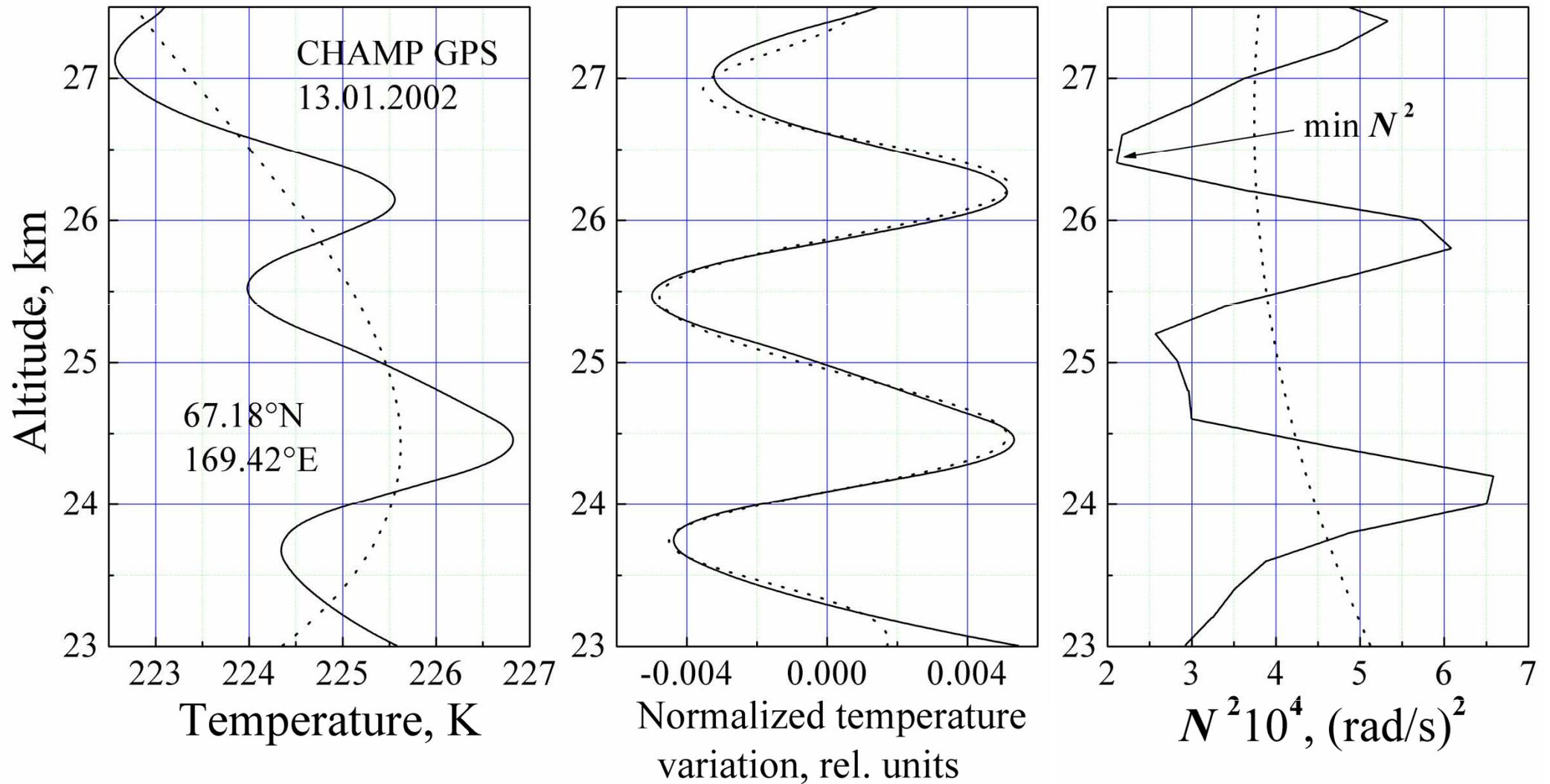
Figure 2. The same as Figure 1, except from radio occultation conducted in the high latitudes of the Earth's atmosphere on January 13, 2002.
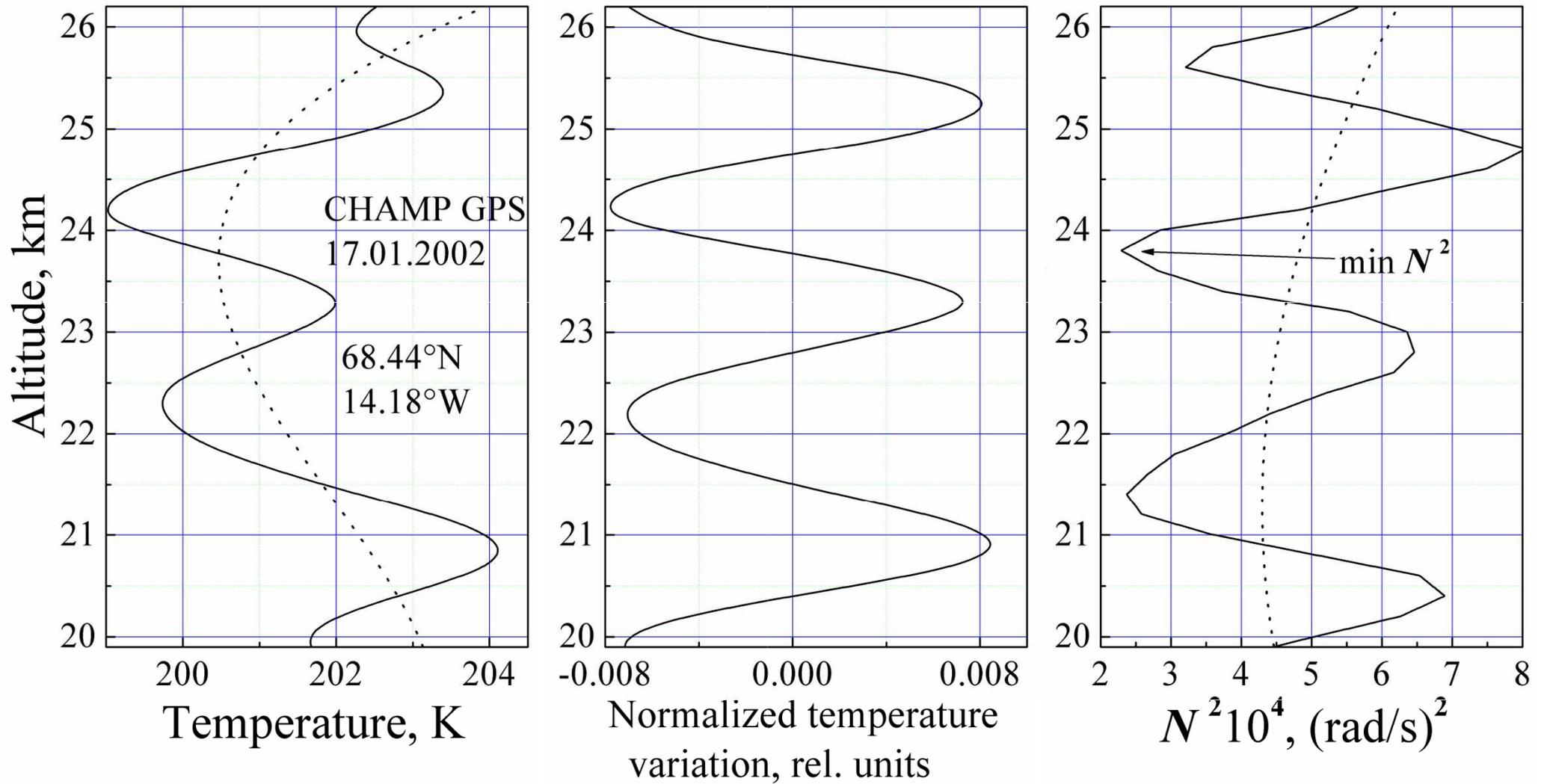
Figure 3. The same as Figure 1, except from radio occultation conducted in the high latitudes of the Earth's atmosphere on January 17, 2002.
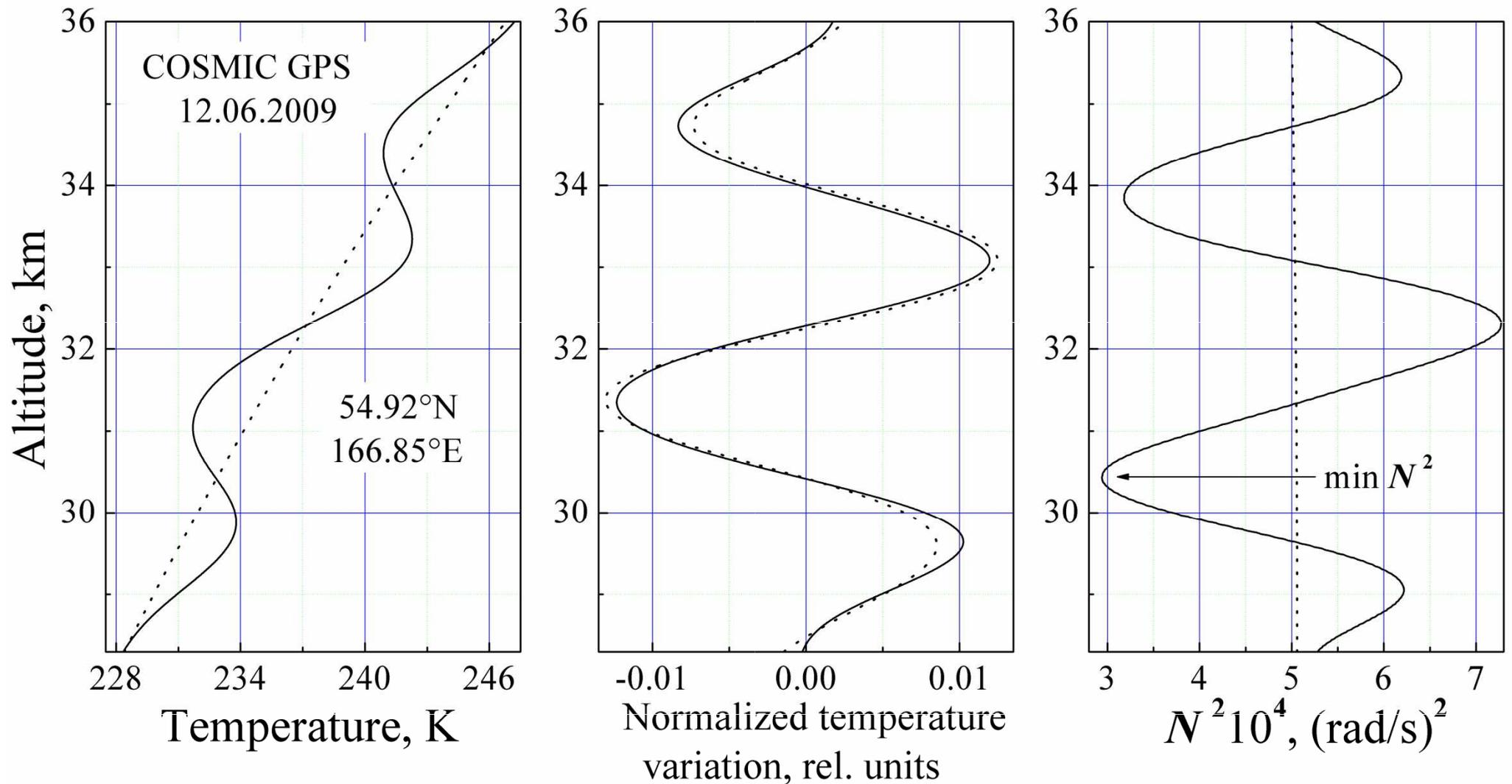
Figure 4. Example of IGW observations in the middle latitudes of the Earth's atmosphere from the temperature $T(h)$ profile measured by COSMIC GPS radio occultation on June 12, 2009. Description of panels is the same as in Figure 1.

Table 2. Wave parameters found from CHAMP & COSMIC GPS radio occultation retrievals of temperature profiles in four regions of the Earth's atmosphere ($\Omega = 7.29 \cdot 10^{-5}$ rad/s is the Earth's rotation rate). The coordinates of sounded regions, time and altitude intervals L of observations are indicated. Uncertainties of determined parameters are also shown if these < 100%

Satellite	CHAMP	CHAMP	CHAMP	COSMIC
IGW parameters	13.01.2002 06h 03m UT 37.09° N 102.96° E [15.5÷24.0] km	13.01.2002 15h 37m UT 67.18° N 169.42° E [23.0÷27.5] km	17.01.2002 15h 27m UT 68.44° N 14.18° W [19.9÷26.2] km	12.06.2009 23h 17m UT 54.92° N 166.85° E [28.3÷36.0] km
L, km	8.5	4.5	6.3	7.7
λ_z, km	3.2 ± 0.7	1.7 ± 0.7	2.1 ± 0.7	3.4 ± 0.8
$\left\|\hat{T}\right\|, 10^{-3}$ rel.un.	10.0 ± 1.4	4.9 ± 1.4	7.8 ± 1.4	12.5 ± 1.4
$N_{med}^2, 10^{-4}$ rad^{2}s^{-2}	4.35 ± 0.64	3.75 ± 0.80	4.50 ± 0.96	5.00 ± 0.55
a_e, rel. un.	0.44 ± 0.14	0.48 ± 0.27	0.50 ± 0.22	0.45 ± 0.13
f/ω, rel. un.	0.96 ± 0.03	0.95 ± 0.08	0.94 ± 0.07	0.96 ± 0.03

ω, 10^{-4} rad s^{-1}	0.92 ± 0.03	1.42 ± 0.12	1.44 ± 0.11	1.25 ± 0.04
T^{in}, hours	19.0 ± 0.7	12.3 ± 1.0	12.1 ± 0.9	14.0 ± 0.5
$\tan \varphi'$, rel. un.	800 ± 436	434	437 ± 353	611 ± 319
$90° - \varphi'$, deg	0.07 ± 0.04	0.13	0.13 ± 0.11	0.09 ± 0.05
$\left\|C_{ph}^{in}\right\|$, ms^{-1}	37.4 ± 17.3	16.4 ± 14.2	21.2 ± 14.3	41.3 ± 18.1
$\|u'\|$, m s^{-1}	16.5 ± 4.2	7.9 ± 3.8	10.7 ± 4.0	18.7 ± 4.8
$\|v'\|$, m s^{-1}	15.9 ± 4.3	7.5 ± 3.9	10.1 ± 4.1	17.9 ± 4.8
$\left\|C_{pz}^{in}\right\|$, 10^{-3} ms^{-1}	46.7 ± 10.9	37.8 ± 16.8	48.6 ± 16.4	67.5 ± 16.0
$\|w'\|$, 10^{-3} m s^{-1}	20.7 ± 8.6	18.1 ± 14.0	24.5 ± 14.8	30.6 ± 12.1
λ_h, km	2560 ± 1260	727 ± 680	927 ± 684	2080 ± 970
$\left\|C_{gh}^{in}\right\|$, ms^{-1}	3.0 ± 1.4	1.6 ± 1.4	2.4 ± 1.6	3.6 ± 1.6
$\left\|C_{gz}^{in}\right\|$, 10^{-3} ms^{-1}	3.8 ± 2.7	3.74	5.5	5.8 ± 3.9
E_p, m^{2}s^{-2}	5.5 ± 1.8	1.5 ± 0.9	3.3 ± 1.4	7.5 ± 1.9
E, m^{2}s^{-2}	137 ± 70	30.8 ± 29.8	57.3 ± 43.0	175 ± 89
$p = E_k/E_p$, rel.un.	23.7 ± 15.0	19.2	16.6 ± 15.2	22.3 ± 13.2
$\|F_h\|$, m^{3}s^{-3}	413 ± 286	50	138	621 ± 417
$\|F_z\|$, m^{3}s^{-3}	0.52 ± 0.46	0.12	0.32	1.02 ± 0.86
$\|F_{ph}\|$, m^{2}s^{-2}	0.17 ± 0.08	0.07 ± 0.06	0.13 ± 0.09	0.29 ± 0.13

CONCLUSIONS

1. The new method for the determination of IGW parameters from a single vertical temperature or density profile measurement in a planetary atmosphere has been developed. This method does not require any additional information not contained in the profile and may be used for the analysis of profiles measured by various techniques.

2. The criterion for the IGW identification has been formulated and argued. In the case when this criterion is satisfied then analyzed temperature or density fluctuations can be considered as wave-induced.

3. The experimental verification of the analysis technique proposed was made by using the results of simultaneous temperature and wind velocity *in situ* measurements obtained in a high-resolution balloon experiment [*Cot and Barat*, 1986], where a nearly monochromatic and long-period wave propagating upward in the stratosphere was identified. Using the temperature data only, we reconstructed all the measured wave parameters with uncertainties not larger than 30%.

4.The suggested method is most effective in the case of low intrinsic IGW frequencies when the experimentally determined amplitude threshold a_e appreciably differs from unity.

5.The analysis of radio occultation data on the basis of proposed method gave the possibility to identify the IGW in the Earth's atmosphere and to determine the magnitudes of key wave parameters such as the intrinsic frequency, amplitudes of vertical and horizontal perturbations of the wind velocity, vertical and horizontal wavelengths, intrinsic vertical and horizontal phase (or group) speeds, kinetic and potential energy, vertical fluxes of the wave energy and horizontal momentum.

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