Sounding the Troposphere by LEO-LEO Occultation: A Simulation Retrieval System and Performance Analysis Results

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[1]Done in part while visiting the Danish Meteorological Institute

Inheritance and Requirements

GPS Occultation

- All operational experience in level 2 retrieval gained from GPS occultation
- Unresolved problems: negative-refractivity bias, horizontal gradients, nonphysical retrievals
- Unresolveable issues: wet-dry ambiguity (though no need to resolve for NWP)

To ACE+

- Make a system which is a first draft retrieval system and performance analysis system
- Simulate multipath, diffraction, high-accuracy amplitude
- Simulate systematic and random noise: stability and power issues
- Retrieve physically-constrained geophysical parameters (T, p, p_w, ρ_{cloud}, r_{rain}, O_3, u, gain)

Steps in Performance Analysis

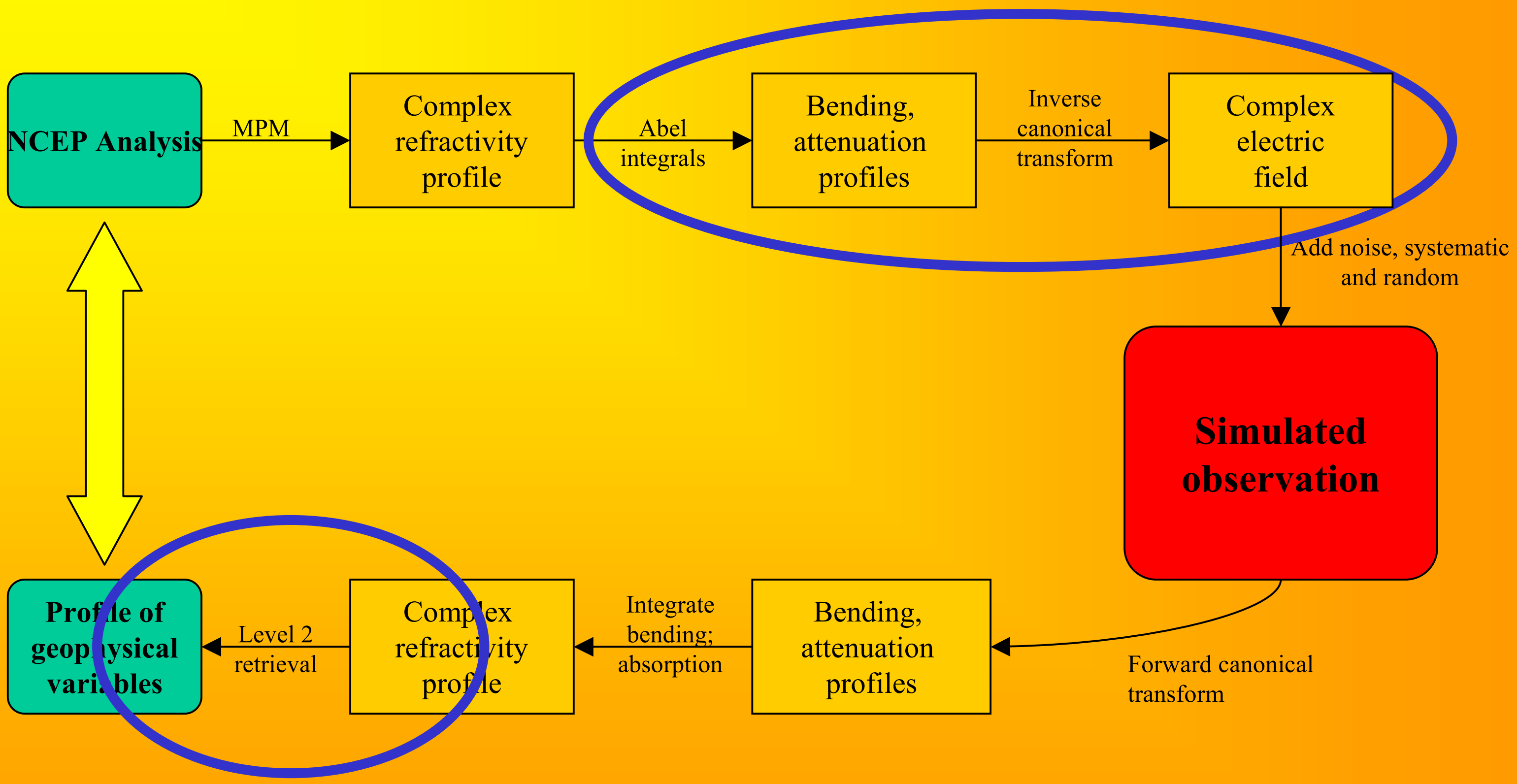
GPS Occultation

1. Atmospheric parameters (T, p, pw) to real index of refraction (*n*)
2. Simulate bending using Abel transform / multiplane diffraction integrals
3. Geometry to simulate phase
4. Superimpose noise on s/c measurement of phase
5. Invert to real index of refraction using geometry / Egorov, then Abel transform
6. Obtain atmospheric parameters by assuming T or p_w or by variational assimilation

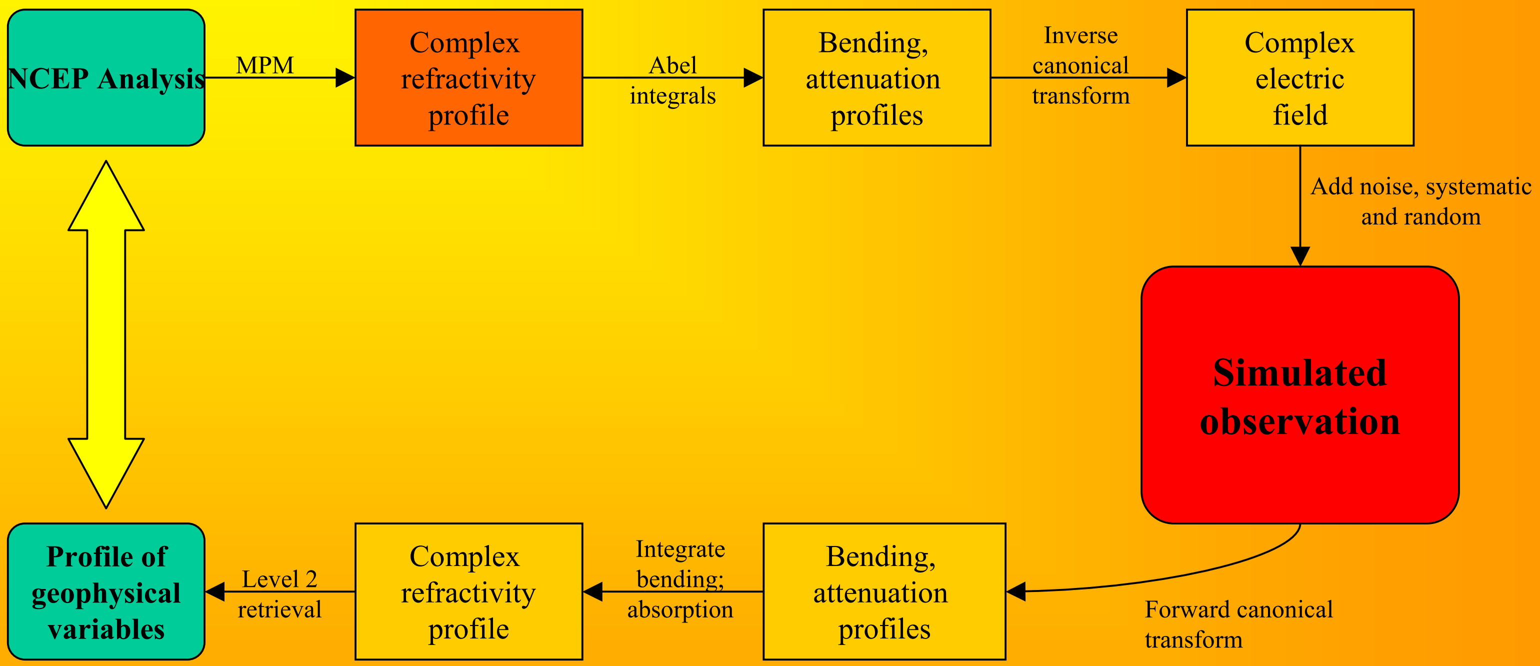
Intersatellite Link Occ'n

1. Atmospheric parameters (T, p, p_w, ρ_{cloud}, r_{rain}, O_3, u) to complex *n*
2. Simulate bending and attenuation using Abel transforms
3. Simulate phase using inverse canonical transform
4. Superimpose noise on receiver measurements of correlation products (open-loop)
5. Invert to complex *n* profiles using forward canonical transform, Abel transforms
6. Obtain physically constrained atmospheric parameters using linearly constrained least squares

The simulation-retrieval system



The simulation-retrieval system



SimDistribution (115 occ'ns)

Properties:

•Multiple transmitters

•Multiple receivers

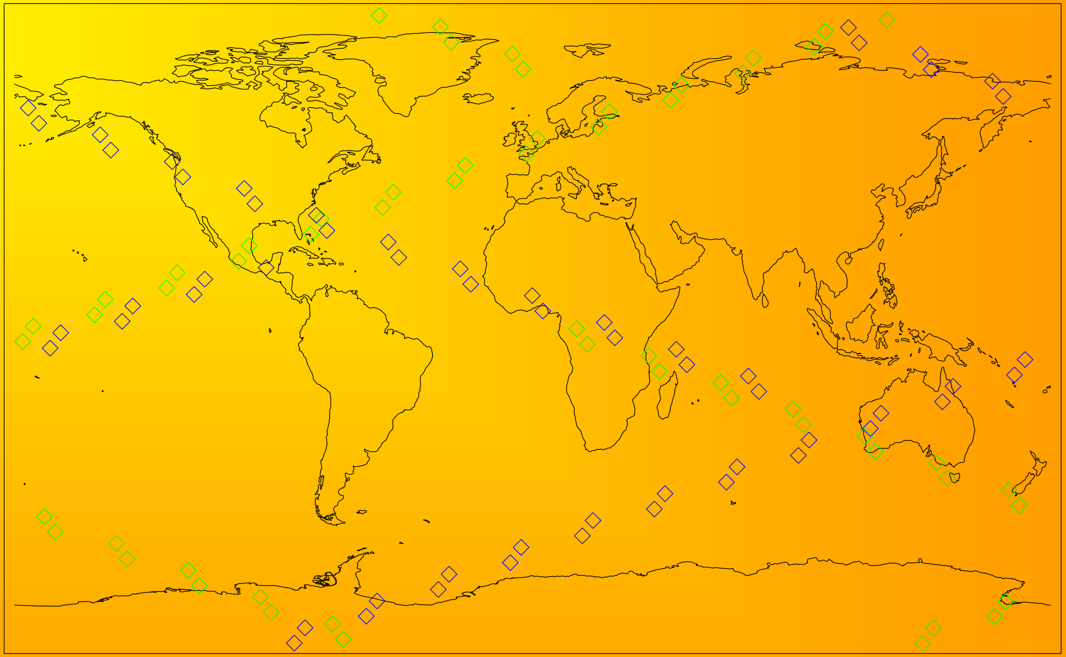
•Mutual occ'ns

•Oblate Earth

•NetCDF output

•Circular orbits

•Variable inclination, altitude, ascending node, anomaly



SimNCEPAtmosphere

Properties

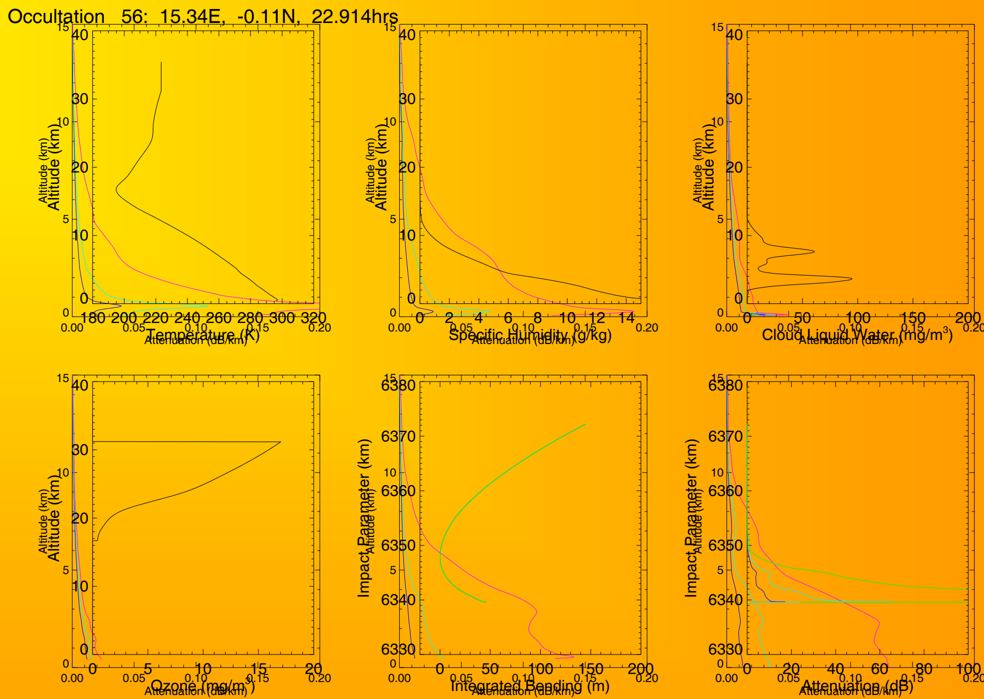
•Uses NCEP analysis starting at any time

•Millimeter-wave Propagation Model

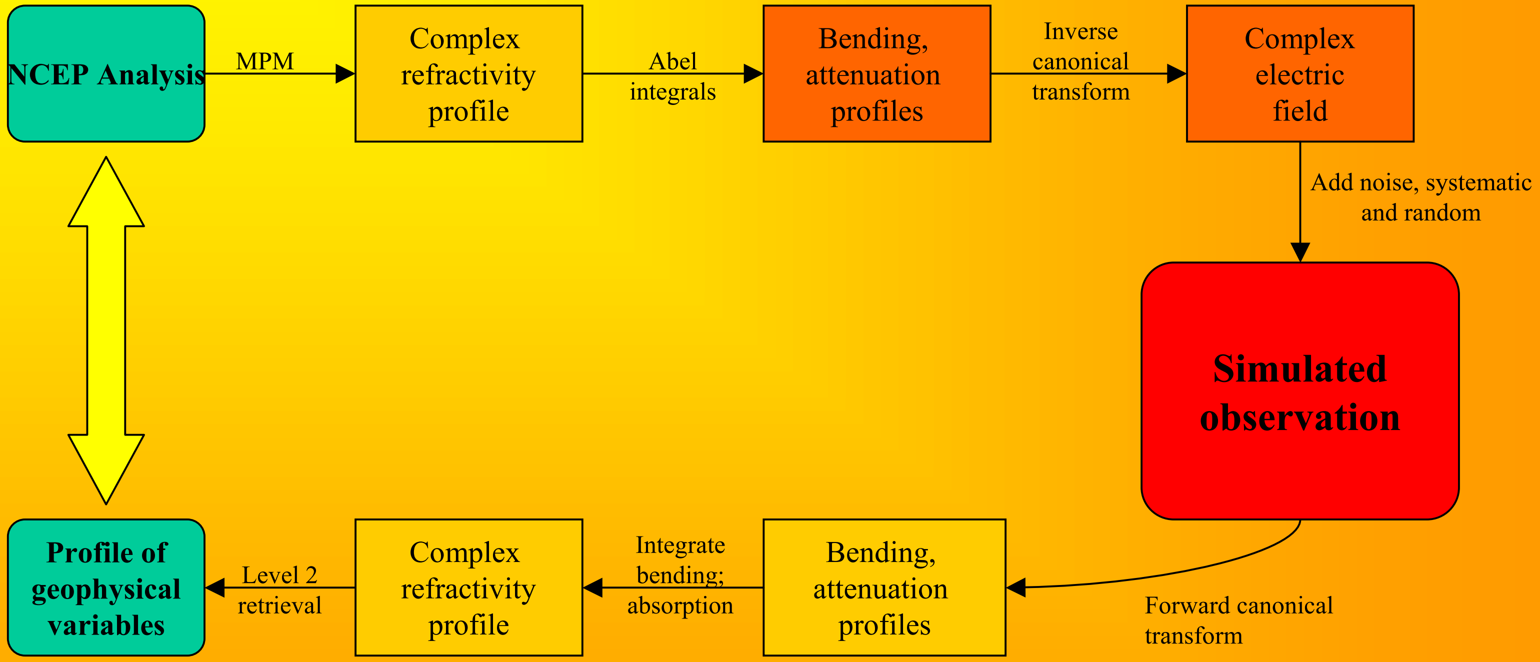
•User-selected frequencies

•Accounts for clouds, rain (future: ozone, wind)

•NetCDF input/output



The simulation-retrieval system



The Inverse Canonical Transform

Methods of simulating observations

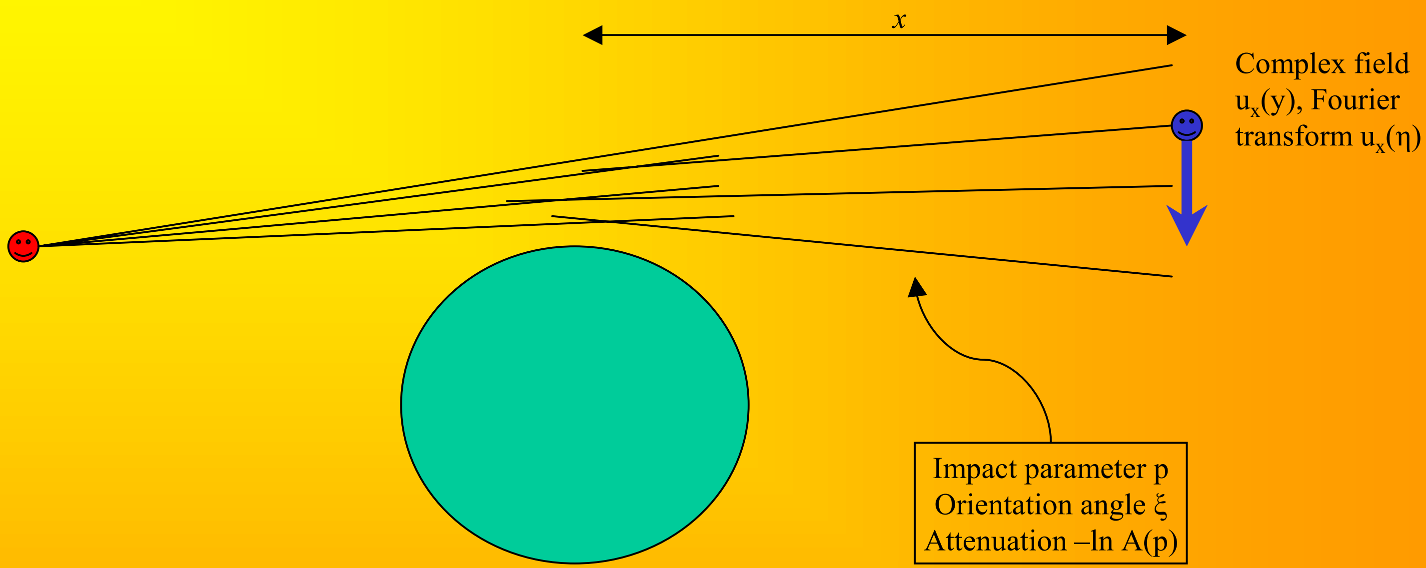
1. Raytracing (1-D, 2-D, 3-D)
   - Ideal for computing phase
   - Difficult to simulate amplitude
   - Handles multipath, but not guaranteed to find all rays
   - Cannot handle diffraction
   - 2-D and 3-D version handle spherically nonsymmetric structures
2. Multiplane (2-D)
   - Computes phase, amplitude very precisely
   - Handles multipath, find all rays
   - Handles diffraction completely
   - Handles along-track asymmetric in atmosphere
   - Very expensive at high frequency

Forward canonical transform

- Given phase and amplitude, compute bending angle and atmospheric attenuation
- 1-Dimensional
- Accounts for multipath and diffraction
- Does not handle spherically nonsymmetric structures
- Computationally inexpensive, two FFTs
- The canonical transform has an analytic inverse which produces phase and amplitude from atmospheric bending and attenuation

(Gorbunov, 2000)

The Canonical Transform



$$\Phi u(p) = A(p)\exp\left[ik\int^{p}\xi(p')\,dp'\right]$$

The Inverse Canonical Transform, cont'd

The forward canonical transform (from Gorbunov) transforms from y,k_y coordinates to p,ξ coordinates:

$$\Phi u_x(p) = \frac{k}{2\pi}\int d\eta\,\left(1-\eta^2\right)^{1/4}\exp\left[ik\left(p\arcsin\eta - x\sqrt{1-\eta^2}\right)\right]\tilde{u}_x(\eta)$$

The inverse canonical transform transforms from p,ξ coordinates to y,k_y :

$$\tilde{u}_x(\eta) = (1-\eta^2)^{-3/4}\, e^{ikx\sqrt{1-\eta^2}}\int \Phi u_x(p)\exp\left[-ikp\arcsin\eta\right]dp$$

Abel Transform Integrals

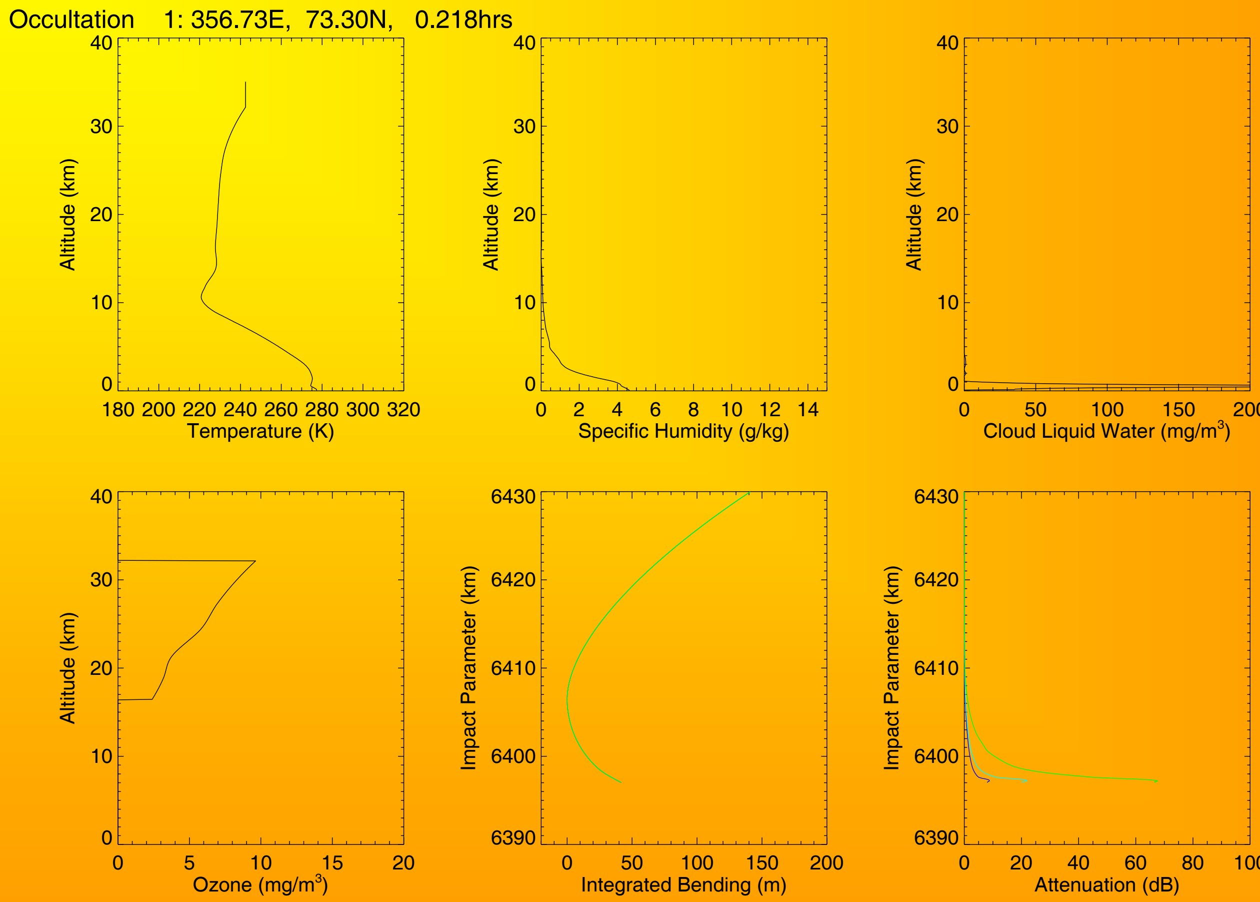
$$\Phi u(p,\upsilon) = A(p,\upsilon)\exp\left[ik\int^{p}\xi(p')\,dp'\right]$$

$$\Phi u(p,\upsilon) = \exp\left\{-2k\int_{r_{\min}}^{\infty}\frac{n_i(\upsilon)\,n_r r\,dr}{\sqrt{(n_r r)^2 - p^2}} - 2ik\int_{r_{\min}}^{\infty}\sqrt{(n_r r)^2 - p^2}\,\frac{d\ln n_r}{dr}\,dr\right\}$$

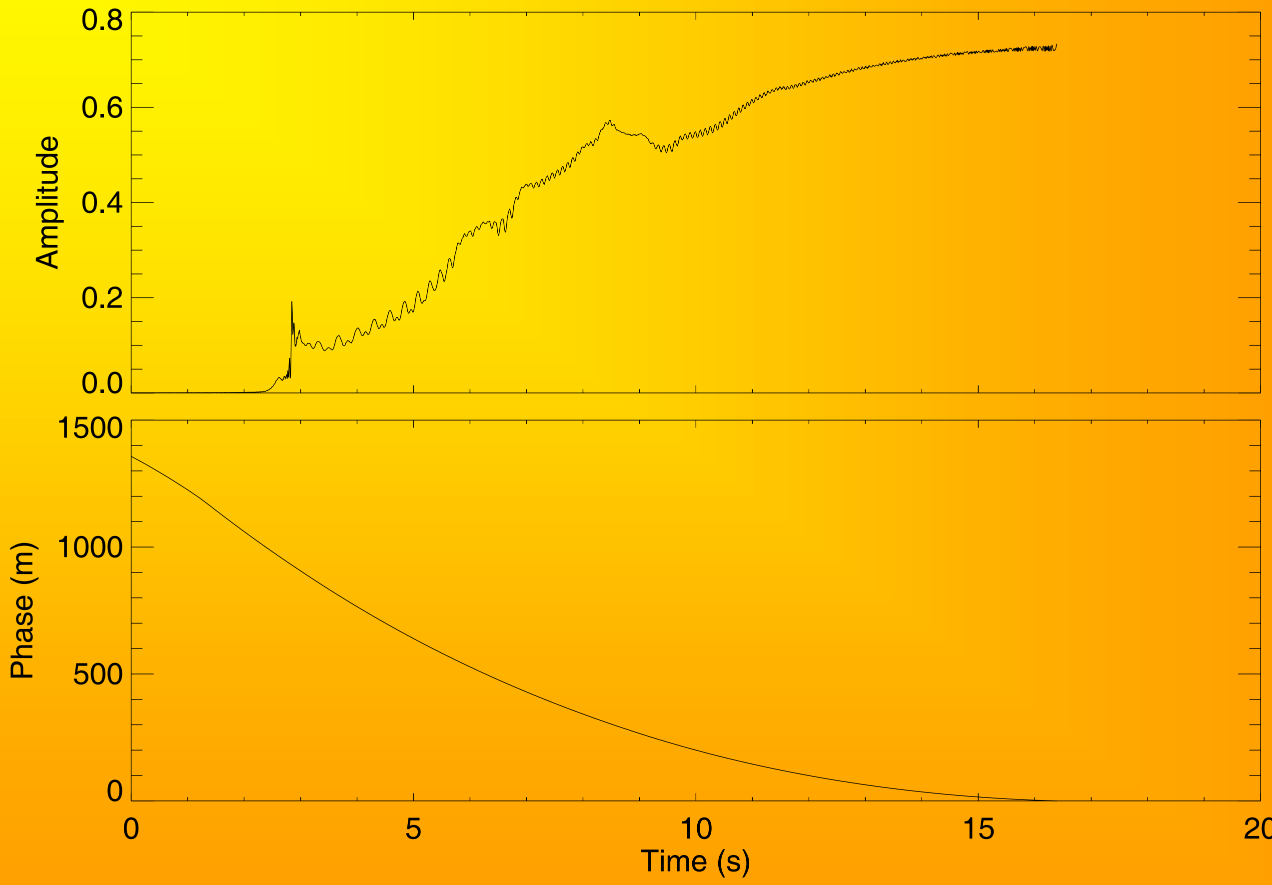
Forward model for attenuation!
(Need corrections for some geometric effects)

Forward model for bending!
(Need correction for incident ray orientation)

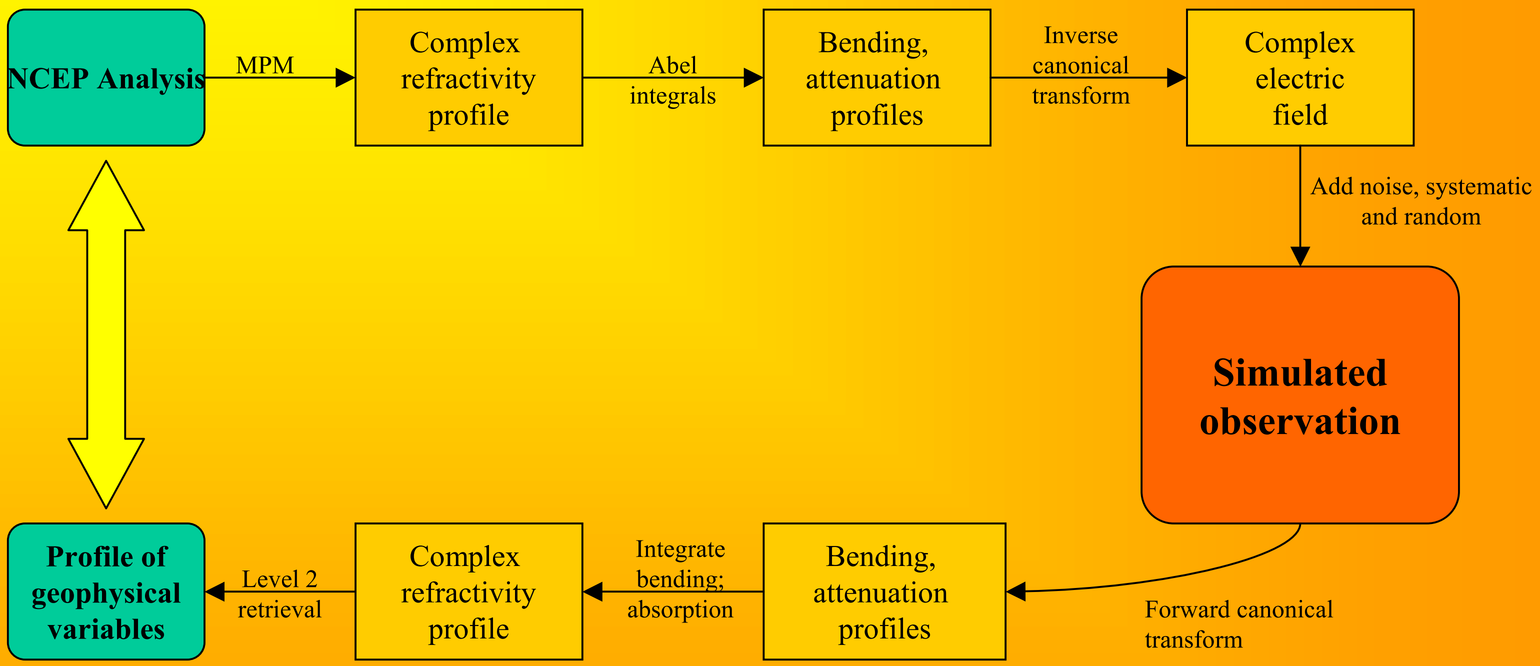
Example simulation



Example simulation, cont'd



The simulation-retrieval system



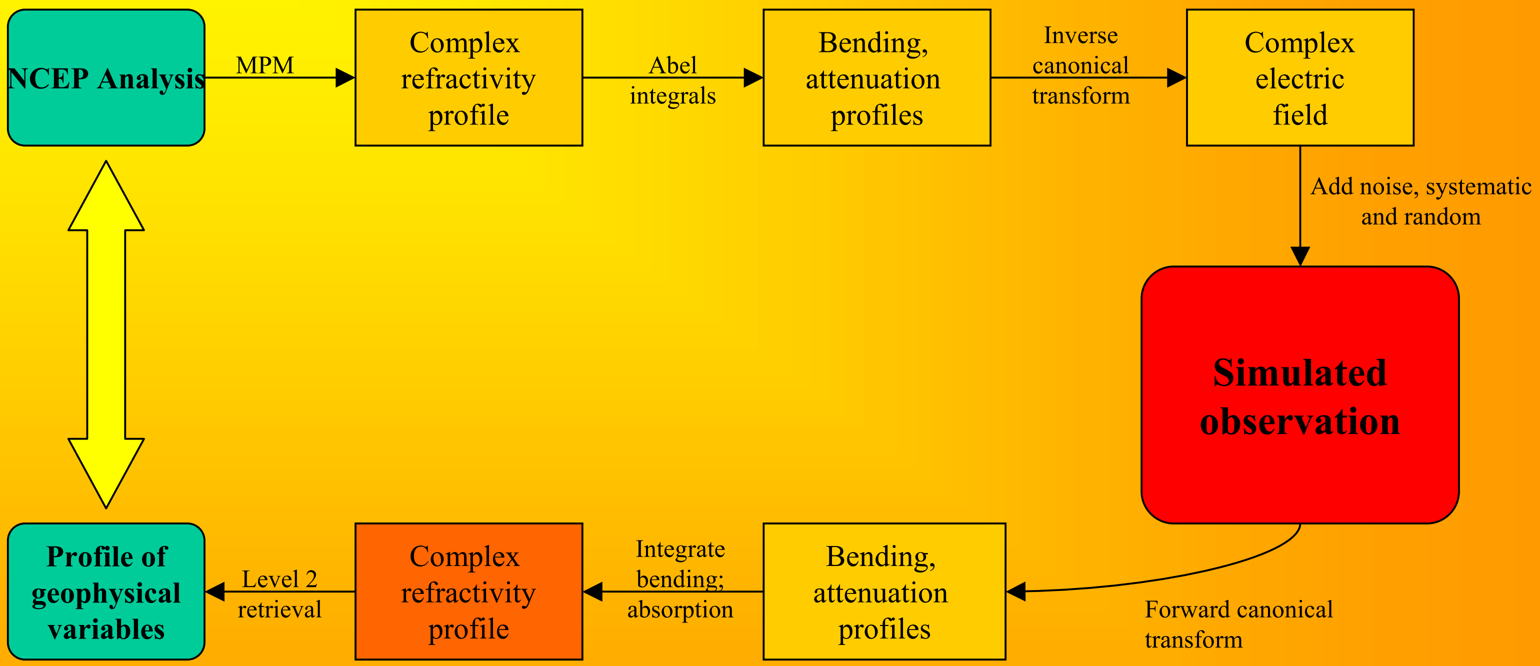
Adding Error

Sources of error:

- Gain drift dg/dt in dB/s
- Signal-to-noise ratio, $\sigma = (v_{s/c}/dy)^{1/2}$ / SNR

$$u_{obs}(y) = u_{true}(y) \cdot 10^{\dot{g}t/10} + \sigma(n_1 + i n_2)$$

The simulation-retrieval system



Abelian inversions

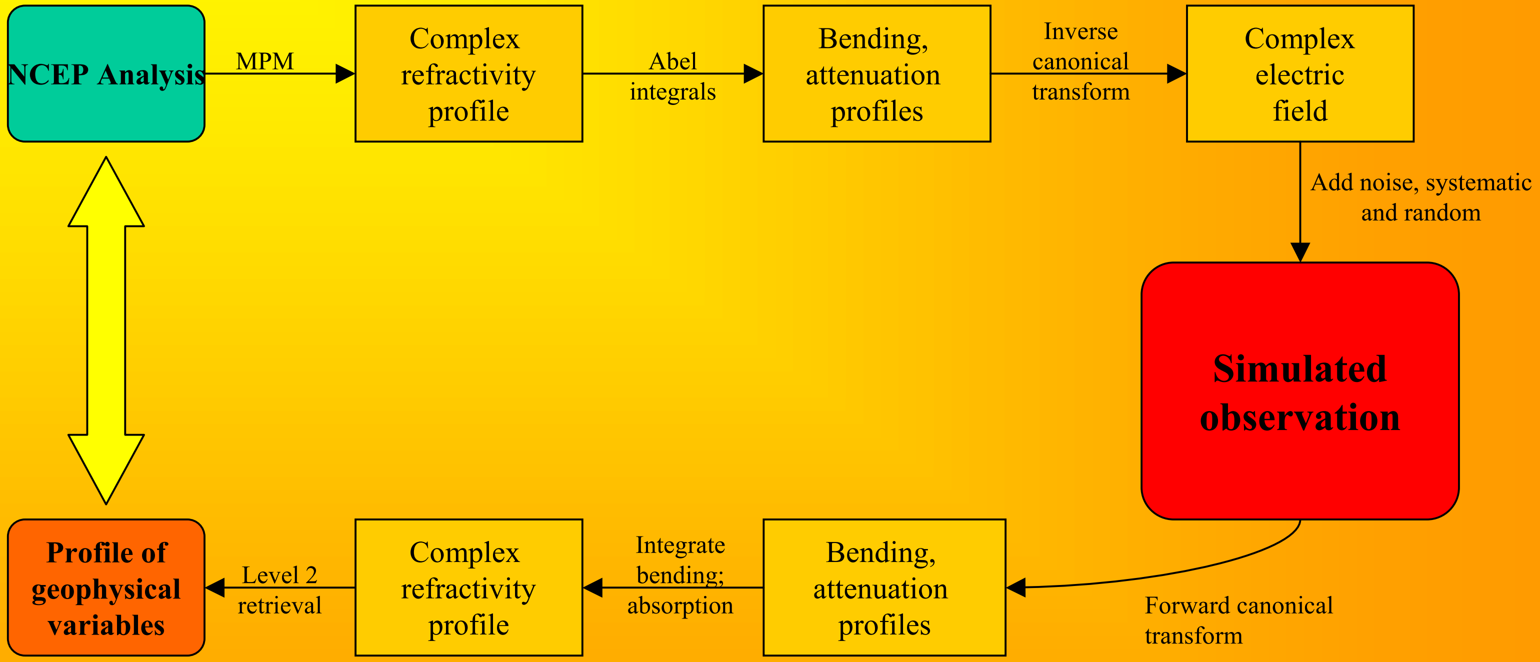
Inversion for real part of refractivity

$$\ln n_r(p) = \frac{1}{\pi}\int_p^{\infty} \frac{\varepsilon(p')\,dp'}{\sqrt{p'^2 - p^2}}$$

Inversion for imaginary part of refractivity

$$n_i(p) = \frac{1}{\pi k p}\frac{dp}{dr}\frac{d}{dp}\int_p^{\infty} \frac{p'\ln(amp)\,dp'}{\sqrt{p'^2 - p^2}}$$

The simulation-retrieval system



Retrieval of geophysical variables

Minimize χ^2 with respect to data and constraints:

$$\chi^2 = (\mathbf{d} - \mathbf{J}_{MPM}\,\delta\mathbf{x})^T \mathbf{O}^{-1} (\mathbf{d} - \mathbf{J}_{MPM}\,\delta\mathbf{x}) + \lambda \left| \delta\mathbf{H} + \mathbf{J}_H\,\delta\mathbf{x} \right|^2$$

Constraints, such as hydrostatic equilibrium, nonnegative water vapour, etc., must satisfy $\mathbf{H}(\mathbf{x})=0$.

Solution: Factorize

$$\left(\mathbf{J}_H{}^T \mathbf{J}_H\right) = \mathbf{e}_H \boldsymbol{\lambda} \mathbf{e}_H^T$$

Remove the constraint space described by the eigenvectors $\mathbf{e}_H$ from the retrieval space ($\mathbf{J}\mathbf{O}^{-1}\mathbf{J}^T$). Then retrieval can be done for each level, starting from the top of the atmosphere. Same as **linearly constrained least squares**: Minimize

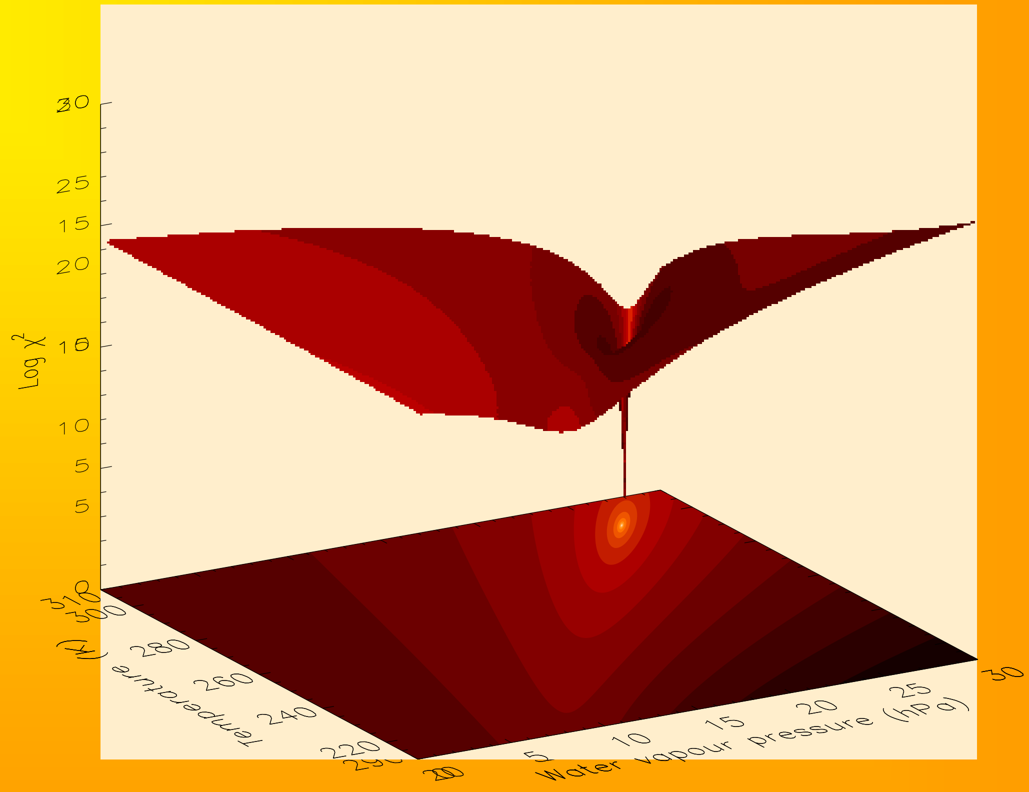
$$\chi^2 = (\mathbf{d} - \mathbf{J}_{MPM}\,\delta\mathbf{x})^T \mathbf{O}^{-1} (\mathbf{d} - \mathbf{J}_{MPM}\,\delta\mathbf{x}) \quad \text{subject to} \quad \delta\mathbf{x} \cdot \nabla\mathbf{H} = 0$$

- Topmost layer: 0DVAR using a climatology of pressure and temperature.
- Subsequent layers: linearly constrained least squares, hydrostatic equilibrium mandatory.
- Added generality: make it possible to retrieve a subset of geophysical variables; use up to 10 frequencies, but subset by frequency and real/imaginary part for each level.

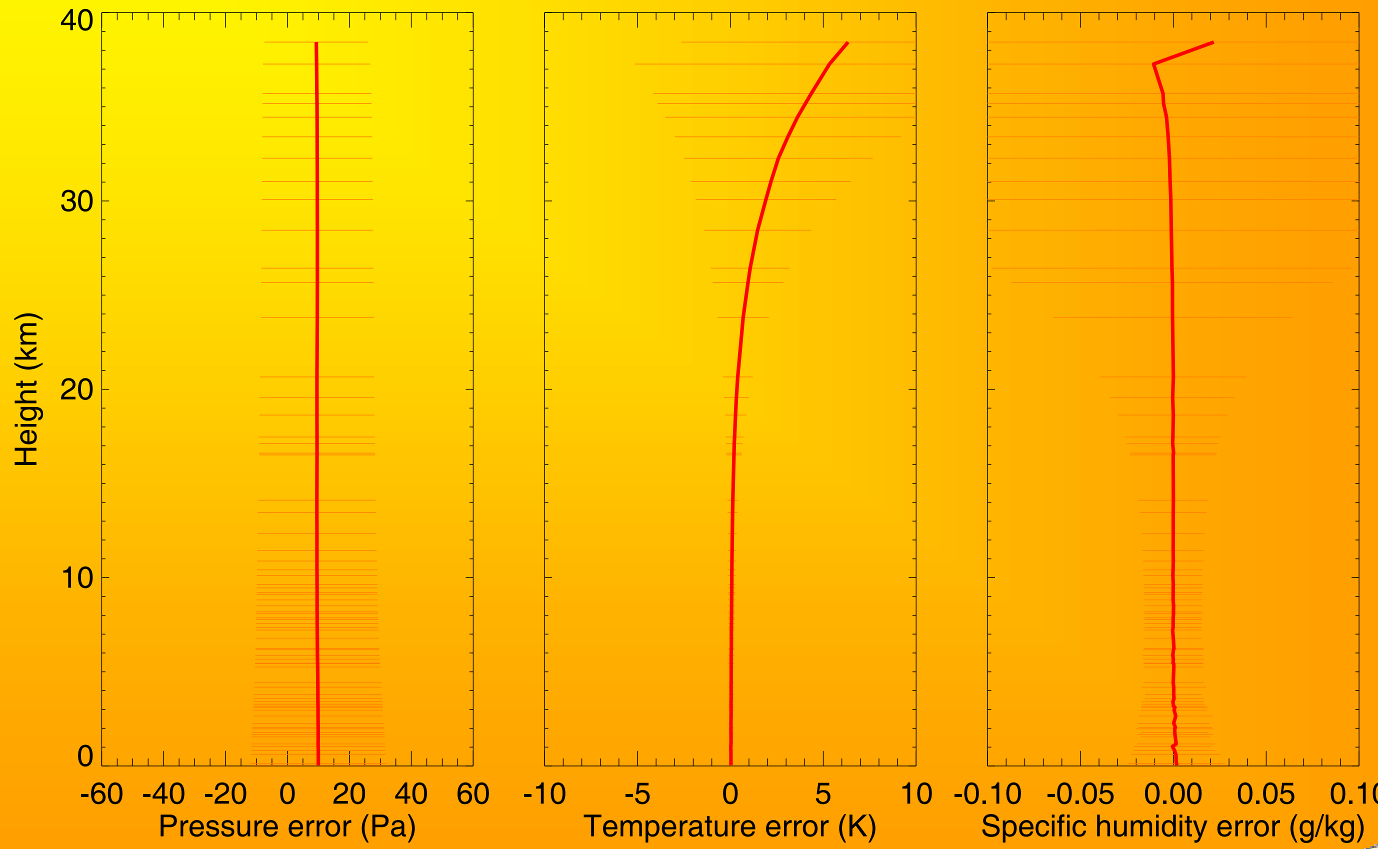
Boundary layer χ^2

Bending and absorption

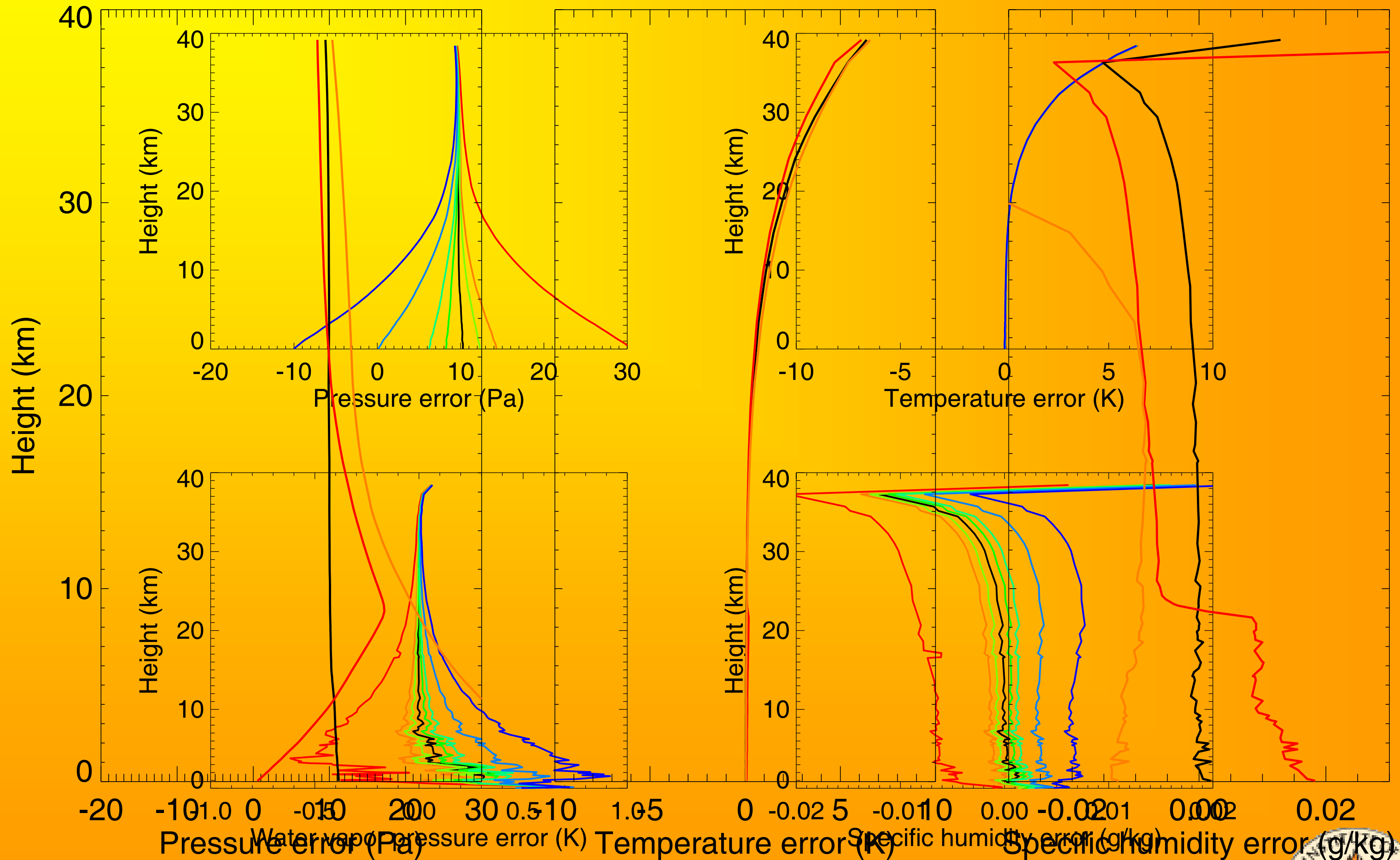
Absorption only



Example level 2 retrieval w/ errors



Example (unrelated) level 2 retrieval



Summary/Work-to-be-done

- Complete forward and inverse canonical transform
- Include line parameterizations for ozone
- Series of runs at 10.3, 17.2, 22.6 GHz
- Series of runs with a "calibration tone" (3-5 GHz)
- Try different gain drifts (0.01-0.10 dB/30s) with and without calibration tone to determine how much the calibration tone helps
- Different SNRs (100-1000) with different sets of frequencies to determine sensitivity in the presence of clouds