Occultations for Probing Atmosphere and Climate

Sept. 16 – 20, Graz, Austria

An inversion approach for the retrieval of atmospheric refraction index profiles from ground-based GPS measurements

Riccardo Notarpietro, Marco Gabella, Giovanni Perona

Politecnico di Torino, Electronics Dept., Turin, Italy

Tel.: +39.011.5644623 Fax: +39.011.5647959 e-mail: rnp@polito.it

OVERVIEW

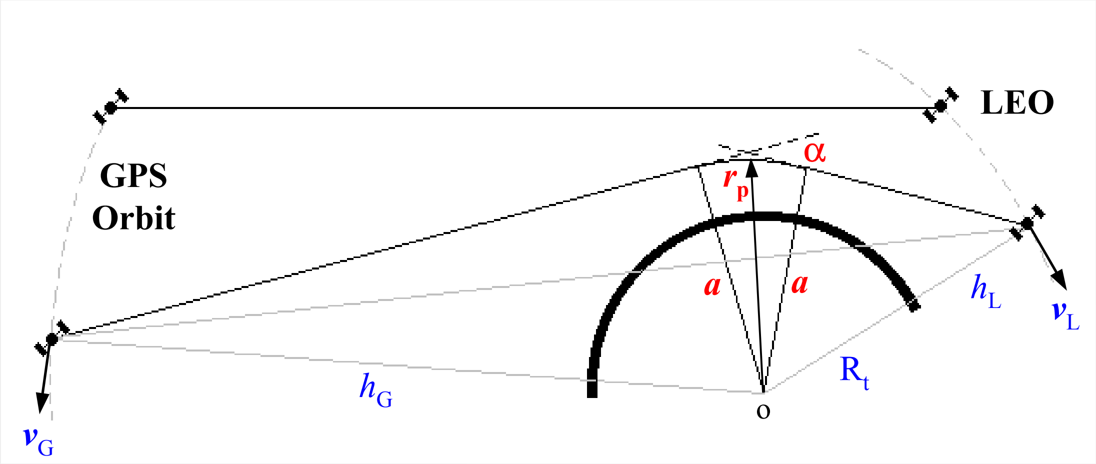
- ✓ Introduction
- ✓ Inapplicability of the Abelian inversion technique
- ✓ GPS observables and methodology adopted for the inversion
- ✓ Arrival angle measurements
- ✓ Results of the inversion
- ✓ Sensitivity analysis
- ✓ Conclusion and outlooks

AIM OF THE RESEARCH

- **Characterization of refraction index profiles for well-mixed atmospheres**
  - ⇒ through the inversion of Earth-Based GPS measurements received at low elevation angles
  - ⇒ adopting a non-linear least squares optimization approach
  - ⇒ for the minimization of a particular cost function

- **Hypothesis**:
  - ⇒ spherical symmetry distribution of atmospheric parameters
  - ⇒ well-mixed atmosphere (anomalous propagation problems have not been taken into account)

Inversion of satellite-based GPS radio-occultation measurement



a: impact parameter

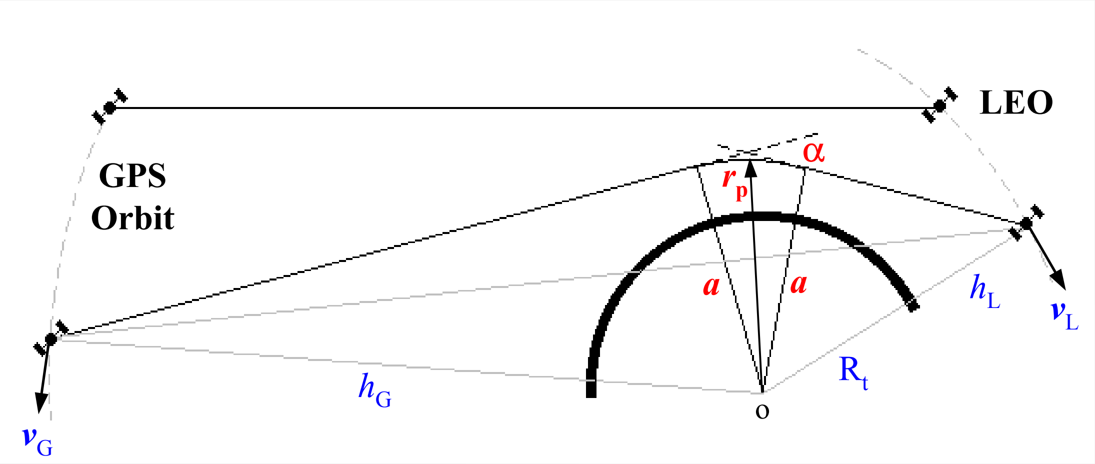
α: total bending angle

$r_{p:}$ tangent radius

Analytical integral formulation between trajectory's geometrical parameters and refraction index profile

$$\alpha(a_{r_p}) = -2a_{r_p} \int_{r_p}^{R_t + h_{atm}} \frac{1}{\sqrt{n^2(r)r^2 - a_{r_p}^2}} \frac{1}{n(r)} \frac{dn}{dr} dr$$

Inversion of satellite-based GPS radio-occultation measurement



a: impact parameter

α: total bending angle

r_{p}: tangent radius

Analytical integral formulation between trajectory's geometrical parameters and refraction index profile

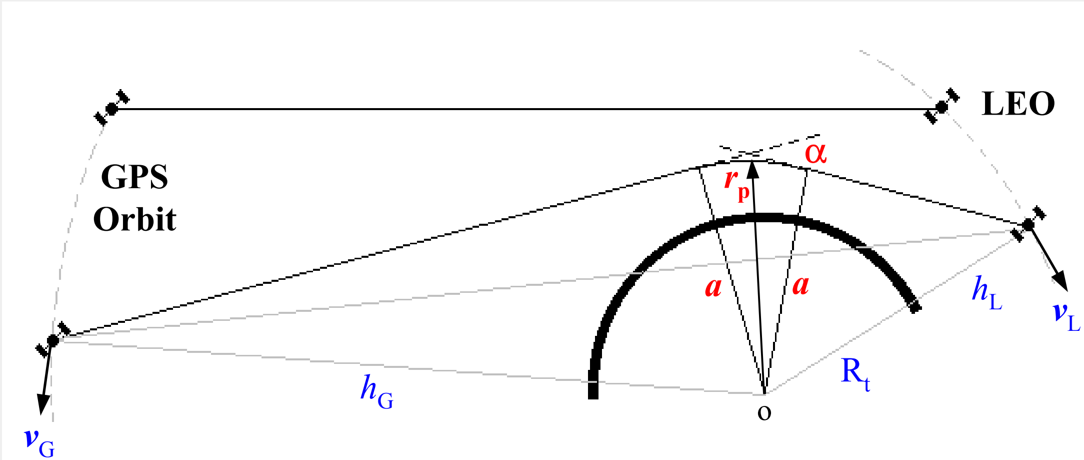
$$\alpha\left(a_{r_p}\right) = -2a_{r_p}\int_{r_p}^{R_t + h_{atm}} \frac{1}{\sqrt{n^2(r)r^2 - a_{r_p}^2}}\frac{1}{n(r)}\frac{dn}{dr}dr$$

Analytical inversion adopting the Abel formulation

$$\ln\left[n\left(r_p\right)\right] = -\frac{1}{\pi}\int_{a_{r_p}}^{R_t + h_{atm}} \ln\left[\frac{a}{a_{r_p}} + \sqrt{\left(\frac{a}{a_{r_p}}\right)^2 - 1}\right]\frac{d\alpha}{da}da$$

and $r_p = \dfrac{a_{r_p}}{n\left(r_p\right)}$

Inversion of satellite-based GPS radio-occultation measurement



a: impact parameter

α: total bending angle

r_p: tangent radius

Analytical integral formulation between trajectory's geometrical parameters and refraction index profile

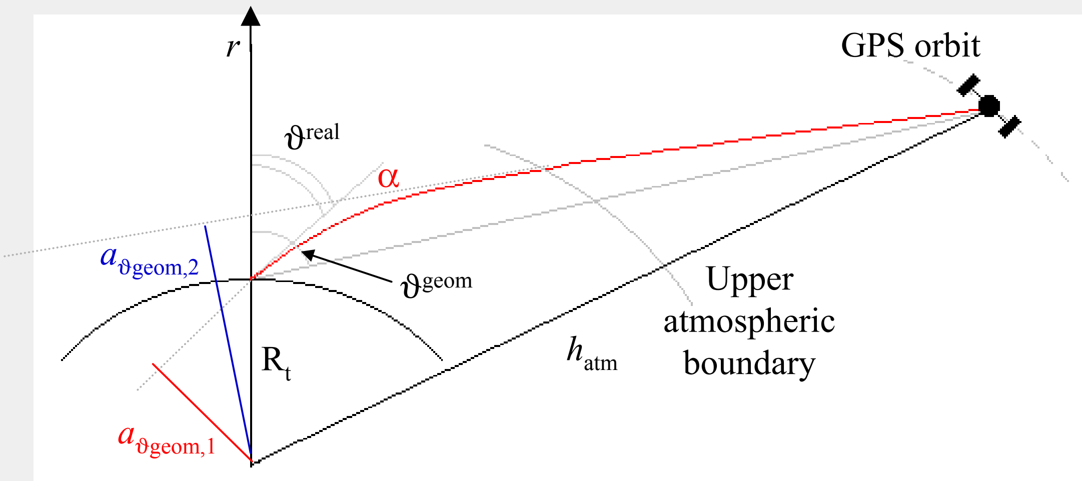
$$\alpha(a_{r_p}) = -2a_{r_p} \int_{r_p}^{R_t + h_{atm}} \frac{1}{\sqrt{n^2(r)r^2 - a_{r_p}^2}} \frac{1}{n(r)} \frac{dn}{dr} dr$$

Analytical inversion adopting the Abel formulation

$$\ln[n(r_p)] = -\frac{1}{\pi} \int_{a_{r_p}}^{R_t + h_{atm}} \ln\left[\frac{a}{a_{r_p}} + \sqrt{\left(\frac{a}{a_{r_p}}\right)^2 - 1}\right] \frac{d\alpha}{da} da$$

and $r_p = \dfrac{a_{r_p}}{n(r_p)}$

Inversion of ground-based GPS measurements



$a_{\vartheta geom,1}$: impact parameter

α: total bending angle

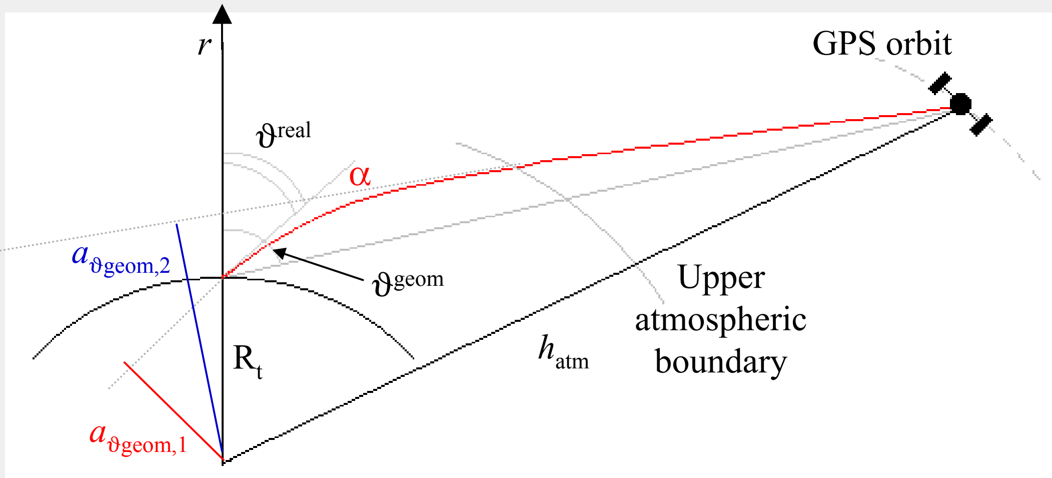
ϑ^{real}: signal arrival angle

ϑ^{geom}: satellite elevation

$$\Delta f_D \Rightarrow^{[1]} \alpha(a_{\vartheta geom,1})$$

[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

Inversion of ground-based GPS measurements



$a_{\vartheta geom,1}$: impact parameter

α: total bending angle

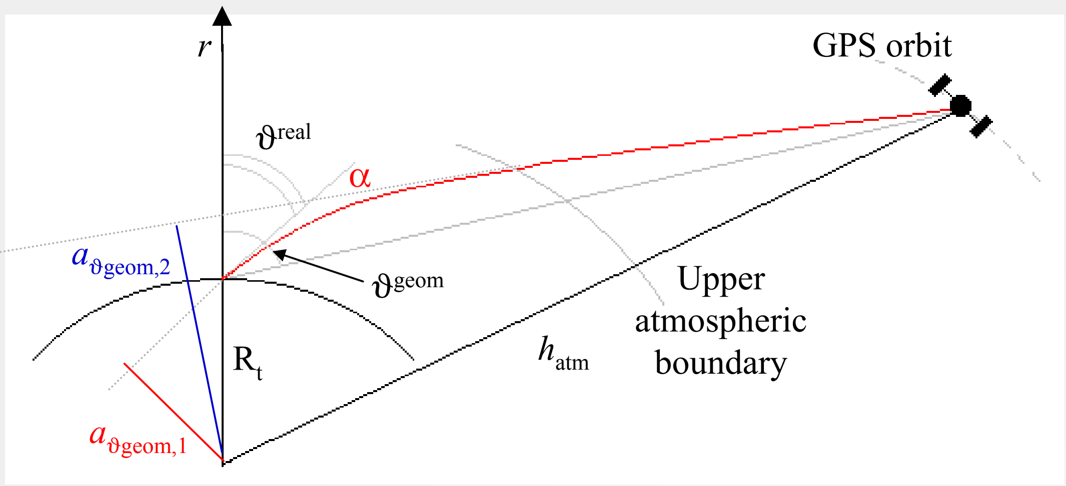
ϑ^{real}: signal arrival angle

ϑ^{geom}: satellite elevation

$$\Delta f_D \Rightarrow^{[1]} \alpha(a_{\vartheta geom,1}) \quad \text{and} \quad \alpha\left(a_{\vartheta geom}\right) = -a_{\vartheta geom} \int_{R_t}^{R_t + h_{atm}} \frac{1}{\sqrt{n^2(r) r^2 - a^2_{\vartheta geom}}} \frac{1}{n(r)} \frac{dn}{dr} dr$$

[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

Inversion of ground-based GPS measurements



$a_{\vartheta geom,1}$: impact parameter

α: total bending angle

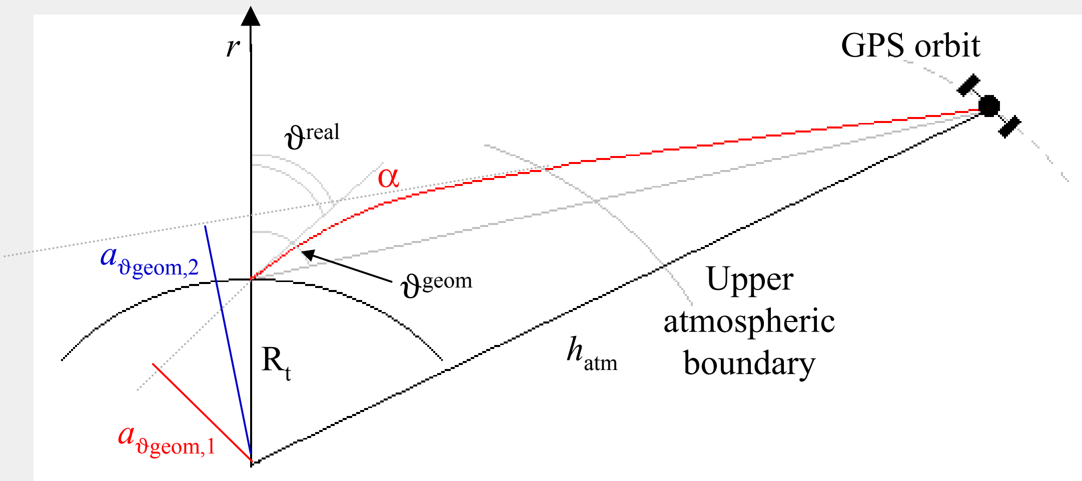
ϑ^{real}: signal arrival angle

ϑ^{geom}: satellite elevation

$$\Delta f_D \Rightarrow^{[1]} \alpha(a_{\vartheta geom,1}) \quad \text{and} \quad \alpha(a_{\vartheta geom}) = -a_{\vartheta geom} \int_{R_t}^{R_t + h_{atm}} \frac{1}{\sqrt{n^2(r) r^2 - a^2_{\vartheta geom}}} \frac{1}{n(r)} \frac{dn}{dr} dr$$

[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

Inversion of ground-based GPS measurements



$a_{\vartheta geom,1}$: impact parameter

α: total bending angle

ϑ^{real}: signal arrival angle

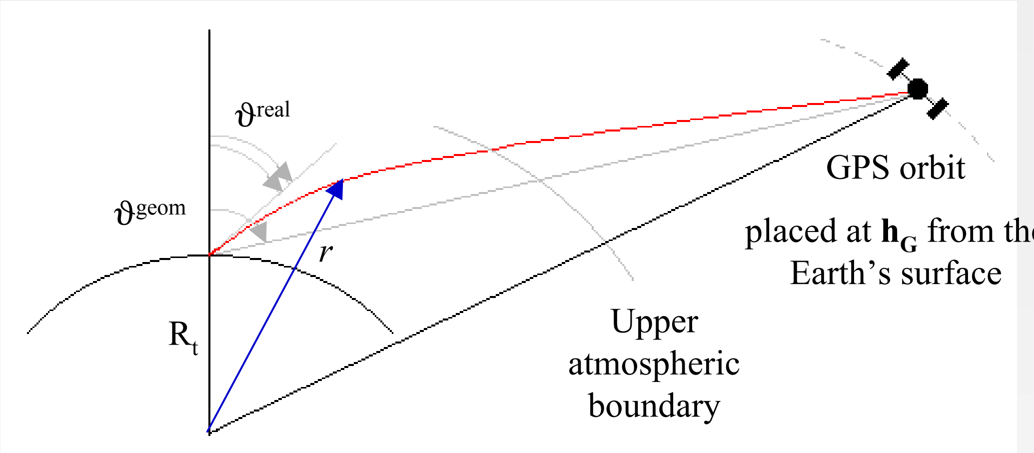
ϑ^{geom}: satellite elevation

$$\Delta f_D \Rightarrow^{[1]} \alpha(a_{\vartheta geom,1}) \quad \text{and} \quad \alpha\left(a_{\vartheta geom}\right) = -a_{\vartheta geom} \int_{R_t}^{R_t + h_{atm}} \frac{1}{\sqrt{n^2(r)r^2 - a_{\vartheta geom}^2}} \frac{1}{n(r)} \frac{dn}{dr} dr$$

To accomplish the inversion, we propose an UNCONSTRAINED OPTIMIZATION approach, based on a NON-LINEAR LEAST SQUARES procedure for the minimization of a particular COST FUNCTION, that depends on the refraction index profile n(r) and on the signal arrival angles ϑ^{real}.

[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

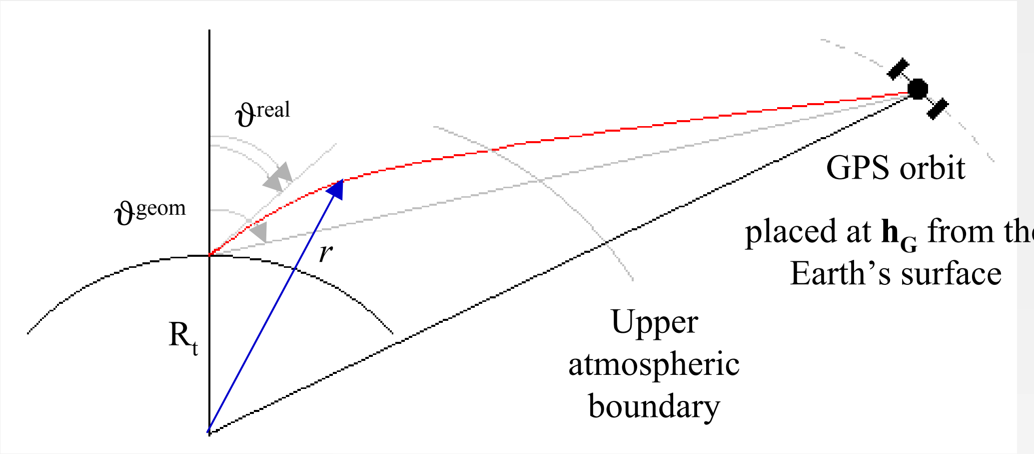
GPS observables



Signal time-of-flight

$$T\left(n(r),\vartheta^{real}\right)=\frac{1}{c}\int_{R_t}^{R_t+h_G}\frac{n(r)}{\sqrt{1-\left[\frac{n(R_t)R_t\sin\left(\vartheta^{real}\right)}{n(r)r}\right]^2}}\mathrm{d}r$$

GPS observables



Signal time-of-flight

$$T\left(n(r),\vartheta^{real}\right)=\frac{1}{c}\int_{R_t}^{R_t+h_G}\frac{n(r)}{\sqrt{1-\left[\frac{n(R_t)R_t\sin\left(\vartheta^{real}\right)}{n(r)r}\right]^2}}\mathrm{d}r$$

Signal arrival angle ϑ^{real} (using the techniques described in the following)

Inversion methodology – COST FUNCTION

Let $\boxed{T_j^{simul}}$ the M values of observables simulated adopting the atmospheric model to be retrieved for the arrival angles ϑ_j^{real} (j = 1, …, M)

Inversion methodology – COST FUNCTION

Let T_j^{simul} the M values of observables simulated adopting the atmospheric model to be retrieved for the arrival angles ϑ_j^{real} (j = 1, …, M)

Let $\hat{T}\left(\mathbf{n}_k, \vartheta_j^{real}\right)$ their estimates, evaluated for the refraction index profile $\mathbf{n}_k$ obtained at the generic algorithm iteration

Inversion methodology – COST FUNCTION

Let T_j^{simul} the M values of observables simulated adopting the atmospheric model to be retrieved for the arrival angles ϑ_j^{real} (j = 1, …, M)

Let $\hat{T}(\mathbf{n}_k, \vartheta_j^{real})$ their estimates, evaluated for the refraction index profile $\mathbf{n}_k$ obtained at the generic algorithm iteration

$$C(\mathbf{n}_k) = \frac{1}{2}\sum_{j=1}^{M}\left|T_j^{simul} - \hat{T}(\mathbf{n}_k, \vartheta_j^{real})\right|^2 = \frac{1}{2}\mathbf{R}^T(\mathbf{n}_k)\mathbf{R}(\mathbf{n}_k)$$

- **non linear**
- **multidimensional**

Residuals vector for the $\mathbf{n}_k$ profile

Inversion methodology – COST FUNCTION

Let T_j^{simul} — the M values of observables simulated adopting the atmospheric model to be retrieved for the arrival angles $\vartheta_j^{\mathrm{real}}$ (j = 1, …, M)

Let $\hat{T}\left(\mathbf{n}_k, \vartheta_j^{\mathrm{real}}\right)$ — their estimates, evaluated for the refraction index profile $\mathbf{n}_k$ obtained at the generic algorithm iteration

$$C(\mathbf{n}_k) = \frac{1}{2}\sum_{j=1}^{M}\left|T_j^{\mathrm{simul}} - \hat{T}\left(\mathbf{n}_k, \vartheta_j^{\mathrm{real}}\right)\right|^2 = \frac{1}{2}\mathbf{R}^T(\mathbf{n}_k)\mathbf{R}(\mathbf{n}_k)$$

- **non linear**
- **multidimensional**

Residuals vector for the $\mathbf{n}_k$ profile

Numerical tools for the iterative progress of the optimization algorithm

$\mathbf{n}_k \Rightarrow$ [$\mathbf{R}(\mathbf{n}_k)$, $\mathbf{J}(\mathbf{n}_k)$] $\Rightarrow \mathbf{n}_{k+1}$

Inversion methodology – COST FUNCTION

Let T_j^{simul} the M values of observables simulated adopting the atmospheric model to be retrieved for the arrival angles ϑ_j^{real} (j = 1, …, M)

Let $\hat{T}\left(\mathbf{n}_k, \vartheta_j^{real}\right)$ their estimates, evaluated for the refraction index profile $\mathbf{n}_k$ obtained at the generic algorithm iteration

$$C(\mathbf{n}_k) = \frac{1}{2}\sum_{j=1}^{M}\left|T_j^{simul} - \hat{T}\left(\mathbf{n}_k, \vartheta_j^{real}\right)\right|^2 = \frac{1}{2}\mathbf{R}^T(\mathbf{n}_k)\mathbf{R}(\mathbf{n}_k)$$

- **non linear**
- **multidimensional**

Residuals vector for the $\mathbf{n_k}$ profile

Numerical tools for the iterative progress of the optimization algorithm

$\mathbf{n}_k \Rightarrow$ [$\mathbf{R}(\mathbf{n}_k)$, $\mathbf{J}(\mathbf{n}_k)$] $\Rightarrow \mathbf{n}_{k+1}$

$$\mathbf{n}_{k+1} = \mathbf{n}_k - \left[\mathbf{J}^T(\mathbf{n}_k)\mathbf{J}(\mathbf{n}_k) + \mu_k\mathbf{I}\right]^{-1}\mathbf{J}^T(\mathbf{n}_k)\mathbf{R}(\mathbf{n}_k)$$

SVD

Inversion methodology – OVERVIEW

Reference atmospheric model → **n_{ref}** → $T^{\text{simul}}\left(\vartheta_j^{\text{real}}\right)$

N unknown values

M measurements

Air Force Reference Atmospheres
mid-latitude July and January model WV neglected

DIRECT PROBLEM

Inversion methodology – OVERVIEW

Reference atmospheric model → $\mathbf{n}_{ref}$ → $T^{simul}\left(\vartheta_j^{real}\right)$

N unknown values — **M** measurements

Air Force Reference Atmospheres
mid-latitude July and January model WV neglected

First guess atmospheric model → $\mathbf{n}_0$ → $\hat{T}\left(\mathbf{n}_0, \vartheta_j^{real}\right)$

Global Bi-exponential model [2]

DIRECT PROBLEM

[2] Kirchengast G., A simple analytical atmospheric model for radio occultation applications, *Tech. Rep. ESA/ESTEC-9/1999*, IMG/UoG, Graz, Austria, 1999.

Inversion methodology – OVERVIEW

Reference atmospheric model → $\mathbf{n}_{\mathrm{ref}}$ → $T^{\mathrm{simul}}\left(\vartheta_j^{\mathrm{real}}\right)$

N unknown values

M measurements

Air Force Reference Atmospheres
mid-latitude July and January model WV neglected

First guess atmospheric model → $\mathbf{n}_0$ → $\hat{T}\left(\mathbf{n}_0, \vartheta_j^{\mathrm{real}}\right)$

Global Bi-exponential model [2]

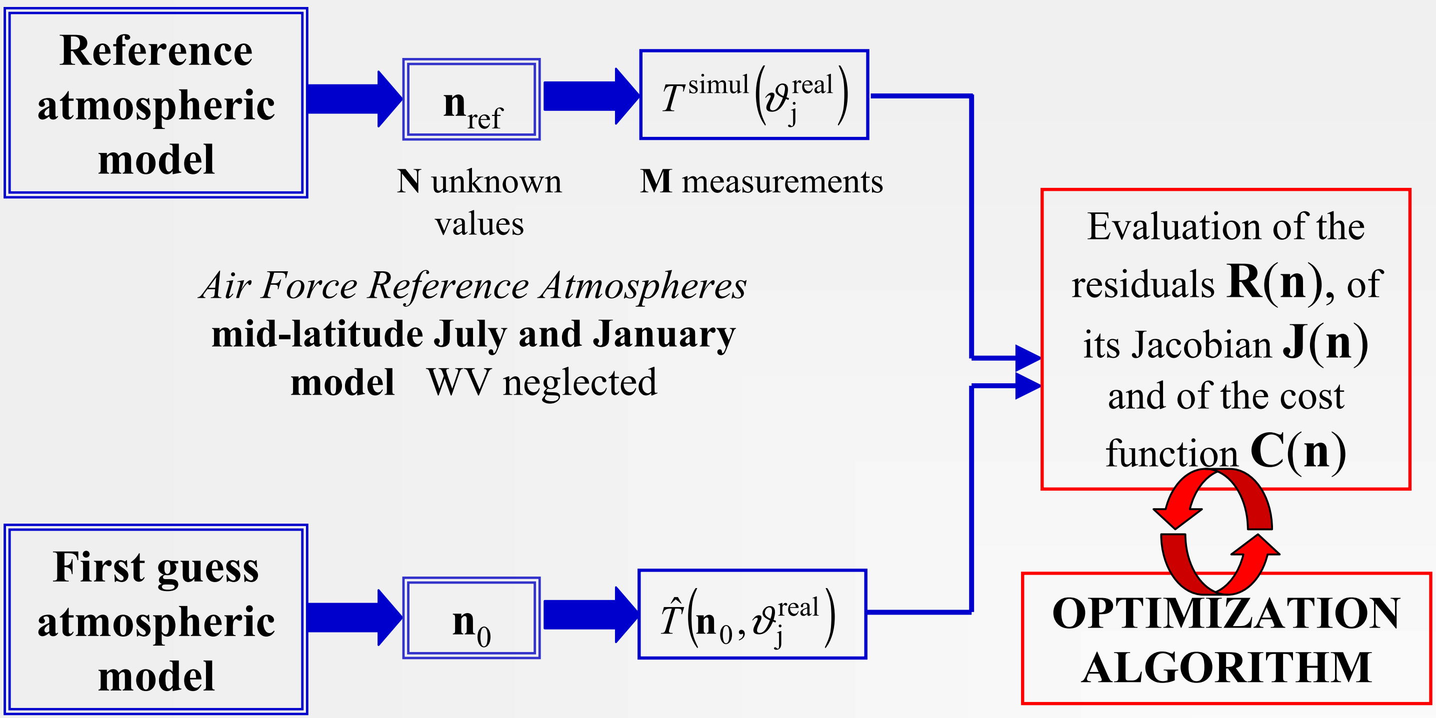
Evaluation of the residuals $\mathbf{R}(\mathbf{n}_0)$, of its Jacobian $\mathbf{J}(\mathbf{n}_0)$ and of the cost function $\mathbf{C}(\mathbf{n}_0)$

DIRECT PROBLEM

INVERSE PROBLEM

[2] Kirchengast G., A simple analytical atmospheric model for radio occultation applications, *Tech. Rep. ESA/ESTEC-9/1999*, IMG/UoG, Graz, Austria, 1999.

Inversion methodology – OVERVIEW



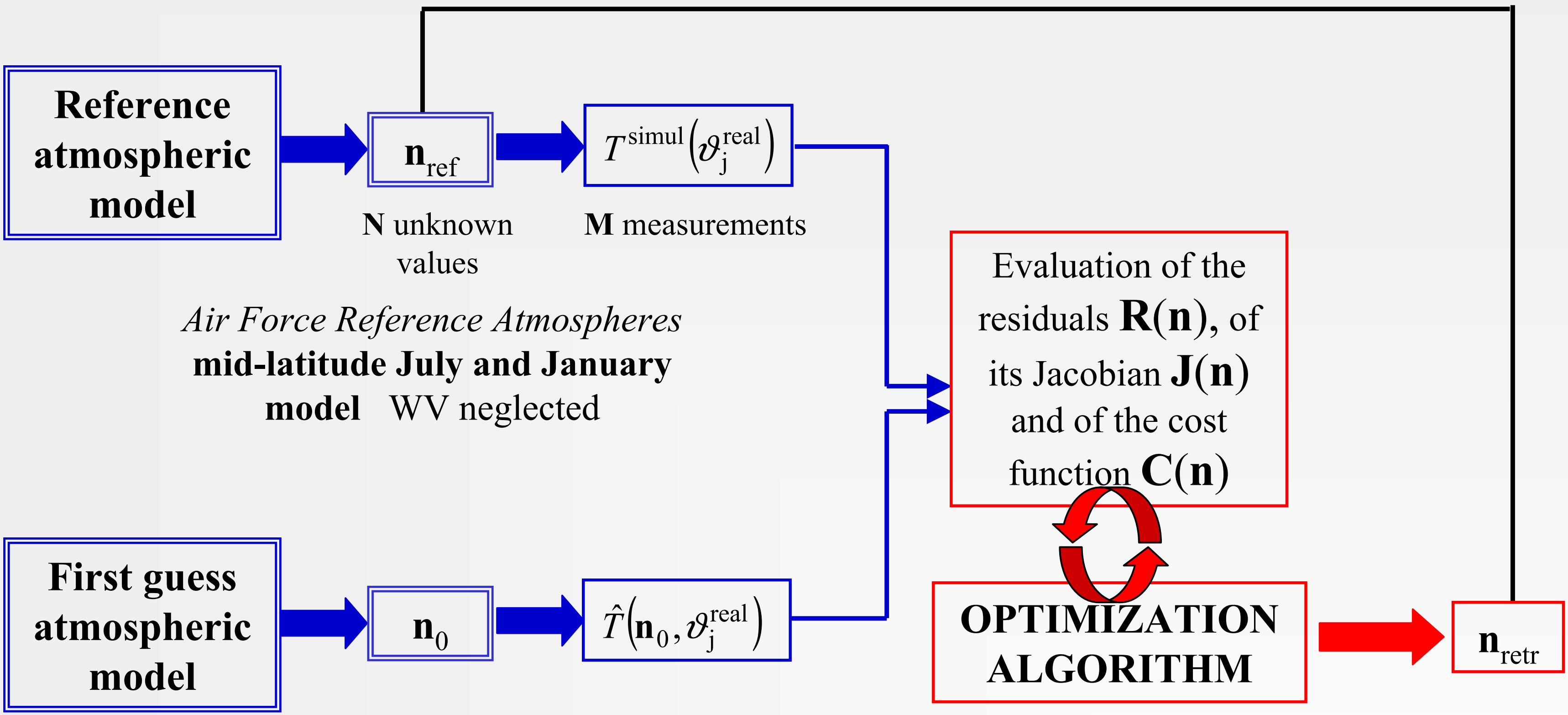
Global Bi-exponential model [2]

INVERSE PROBLEM

DIRECT PROBLEM

[2] Kirchengast G., A simple analytical atmospheric model for radio occultation applications, *Tech. Rep. ESA/ESTEC-9/1999*, IMG/UoG, Graz, Austria, 1999.

Inversion methodology – OVERVIEW



Global Bi-exponential model [2]

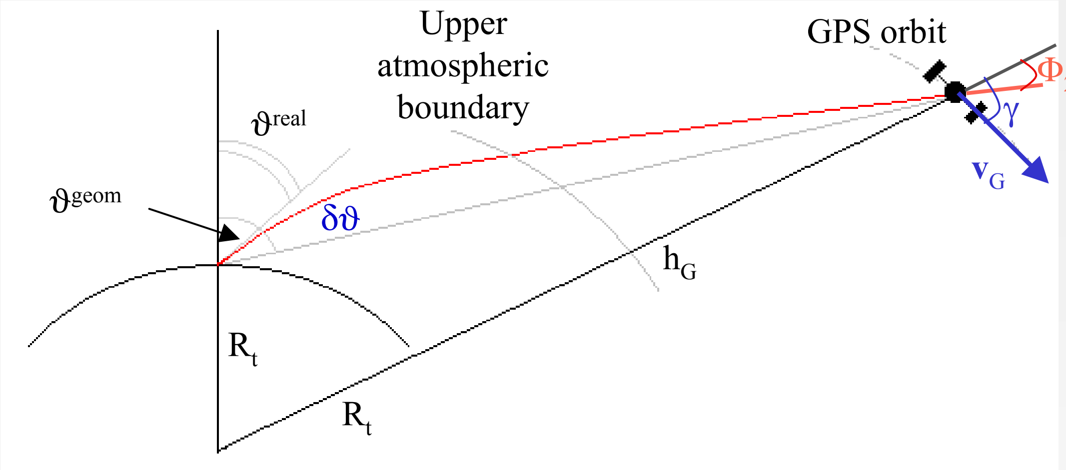
DIRECT PROBLEM

INVERSE PROBLEM

[2] Kirchengast G., A simple analytical atmospheric model for radio occultation applications, *Tech. Rep. ESA/ESTEC-9/1999*, IMG/UoG, Graz, Austria, 1999.

Arrival angles measurements (1)

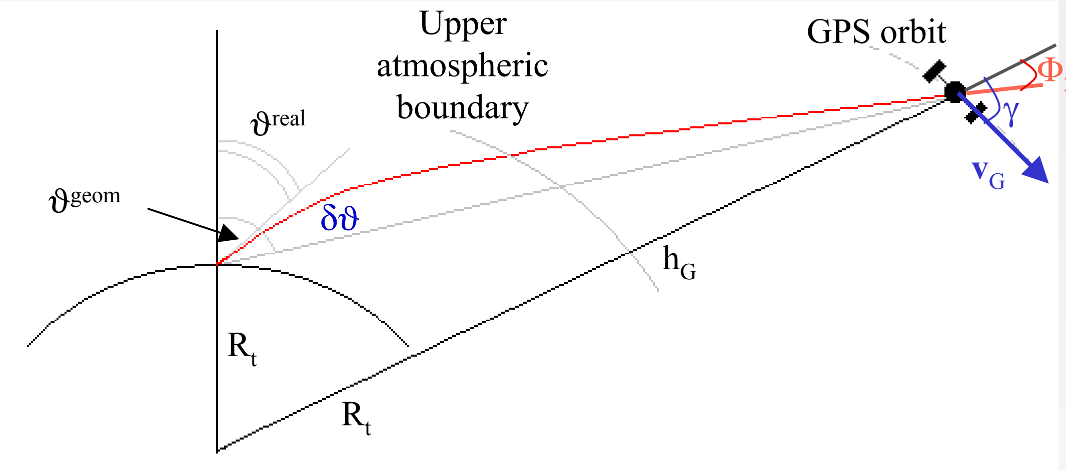
Indirect evaluation through Doppler shift [1]



[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

Arrival angles measurements (1)

Indirect evaluation through Doppler shift [1]



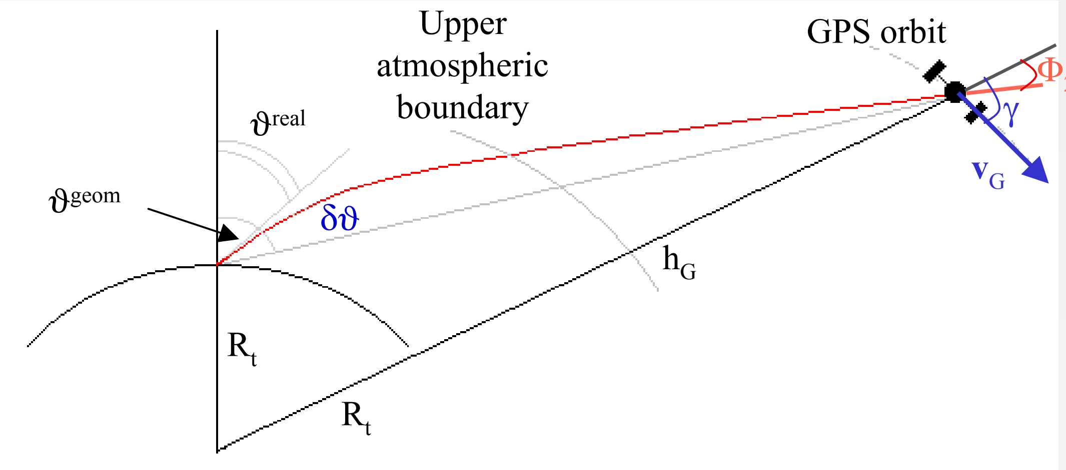
Measuring the <u>Doppler shift</u> in a common Earth-fixed reference frame

$$f_D = -\frac{f}{c}\left|\mathbf{v}_G\right|\cos\left(\gamma - \Phi_2\right)$$

[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

Arrival angles measurements (1)

Indirect evaluation through Doppler shift [1]



Measuring the Doppler shift in a common Earth-fixed reference frame

$$f_D = -\frac{f}{c}\left|\mathbf{v}_G\right|\cos(\gamma - \Phi_2)$$

Using the Snell's low for spherically distributed propagation media

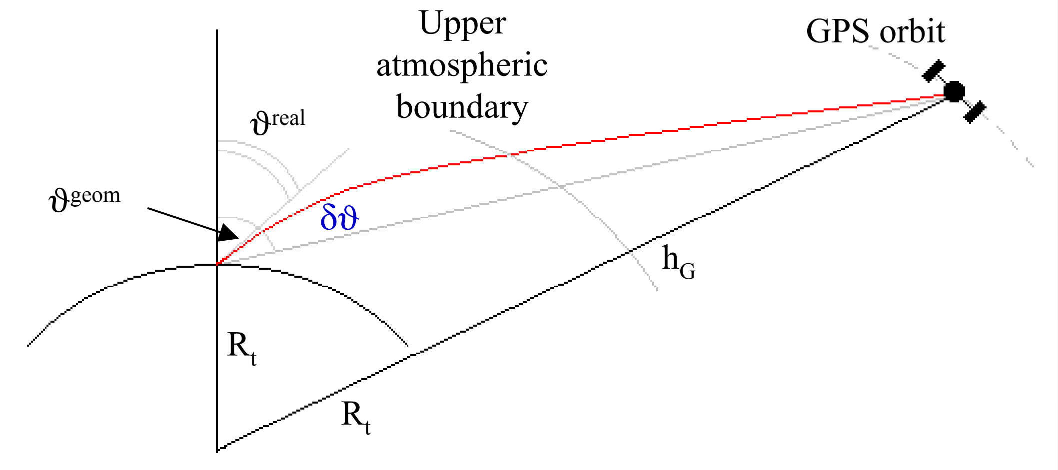
$$n(R_t)R_t\sin(\vartheta_{real}) = (R_t + h_G)\sin(\Phi_2)$$

$\Rightarrow$ ϑ_{real}

[1] Sokolovskiy S.V., C. Rocken, and A. Lowry, Use of GPS for estimation of bending angles of radio waves at low elevations, *Radio Science*, *36*(3), 473-482, 2001.

Arrival angles measurements (2)

Indirect evaluation from phase measurements



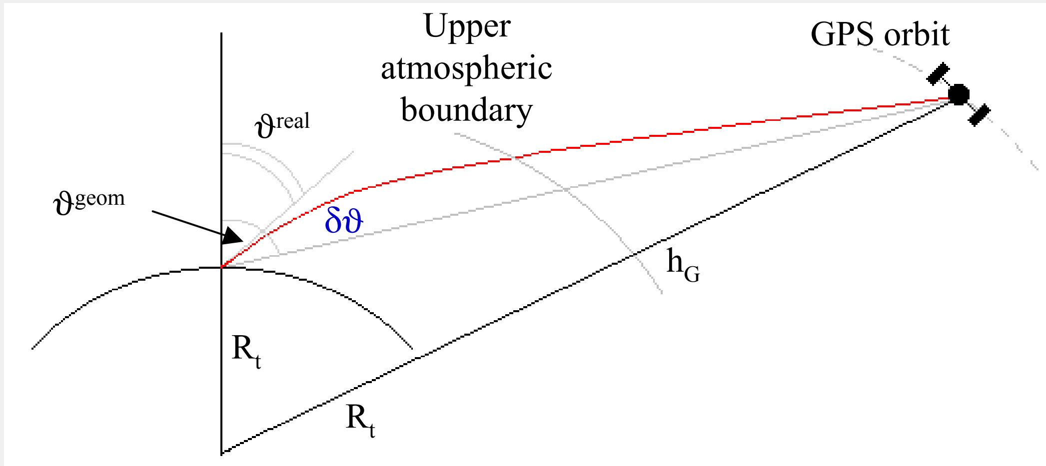
For the July atmospheric model $\delta\vartheta = \vartheta_{geom} - \vartheta_{real}$ is approximately equal to:

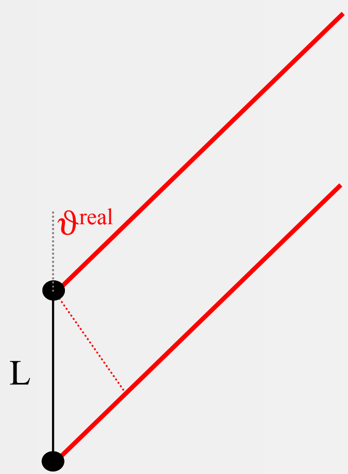
0.6° when $\vartheta_{geom} = 90°$

0.03° when $\vartheta_{geom} = 70°$

Arrival angles measurements (2)

Indirect evaluation from phase measurements

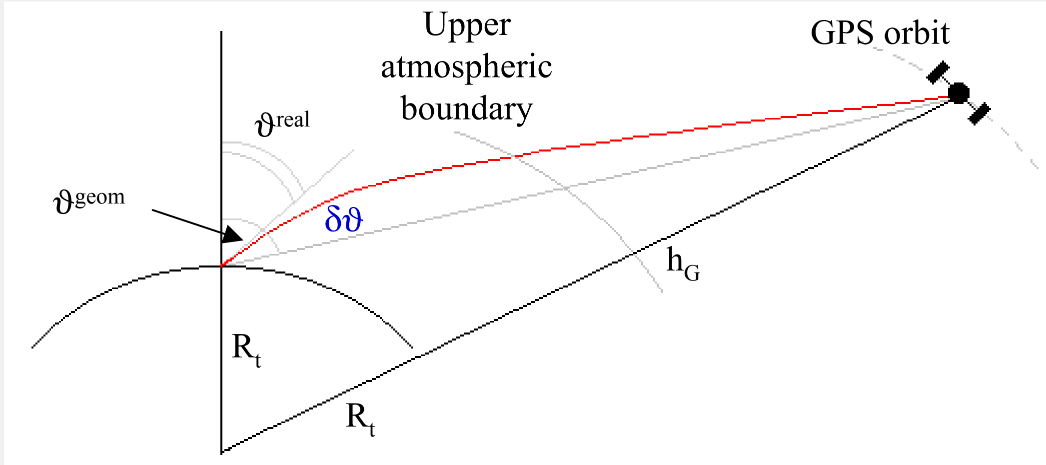


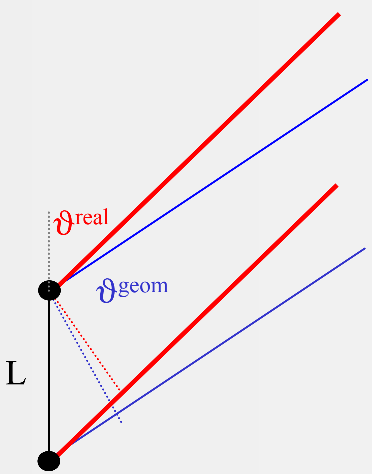


$$\Delta\varphi^{real} = \frac{2\pi}{\lambda} L\cos\left(\vartheta^{real}\right)$$

Arrival angles measurements (2)

Indirect evaluation from phase measurements





$$\Delta\varphi^{real} = \frac{2\pi}{\lambda} L\cos\left(\vartheta^{real}\right)$$

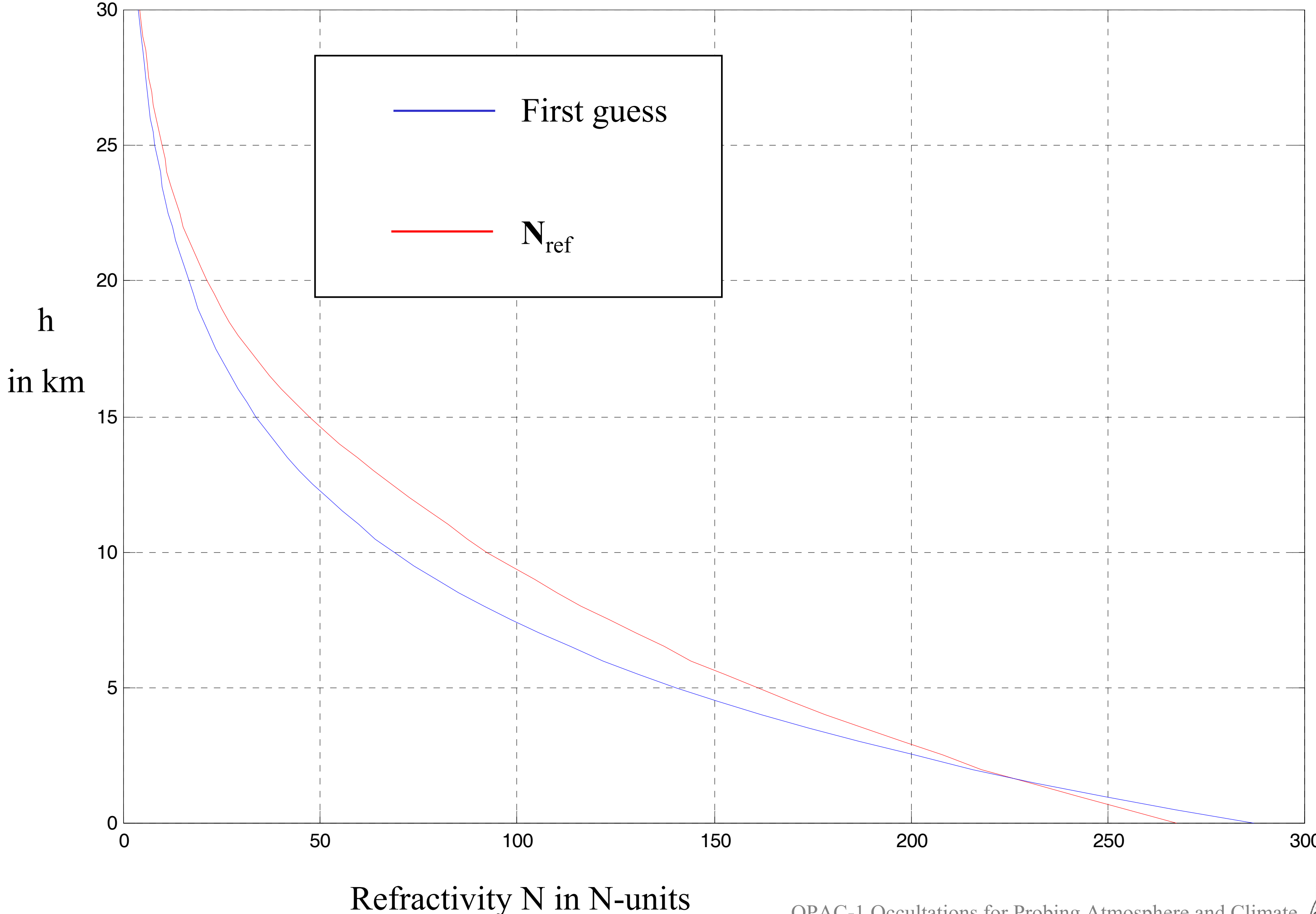
$$\delta(\Delta\varphi) = \Delta\varphi^{real} - \Delta\varphi^{geom}$$

MEASURABLE and < 2π

Results – refractivity profiles

July atmospheric model

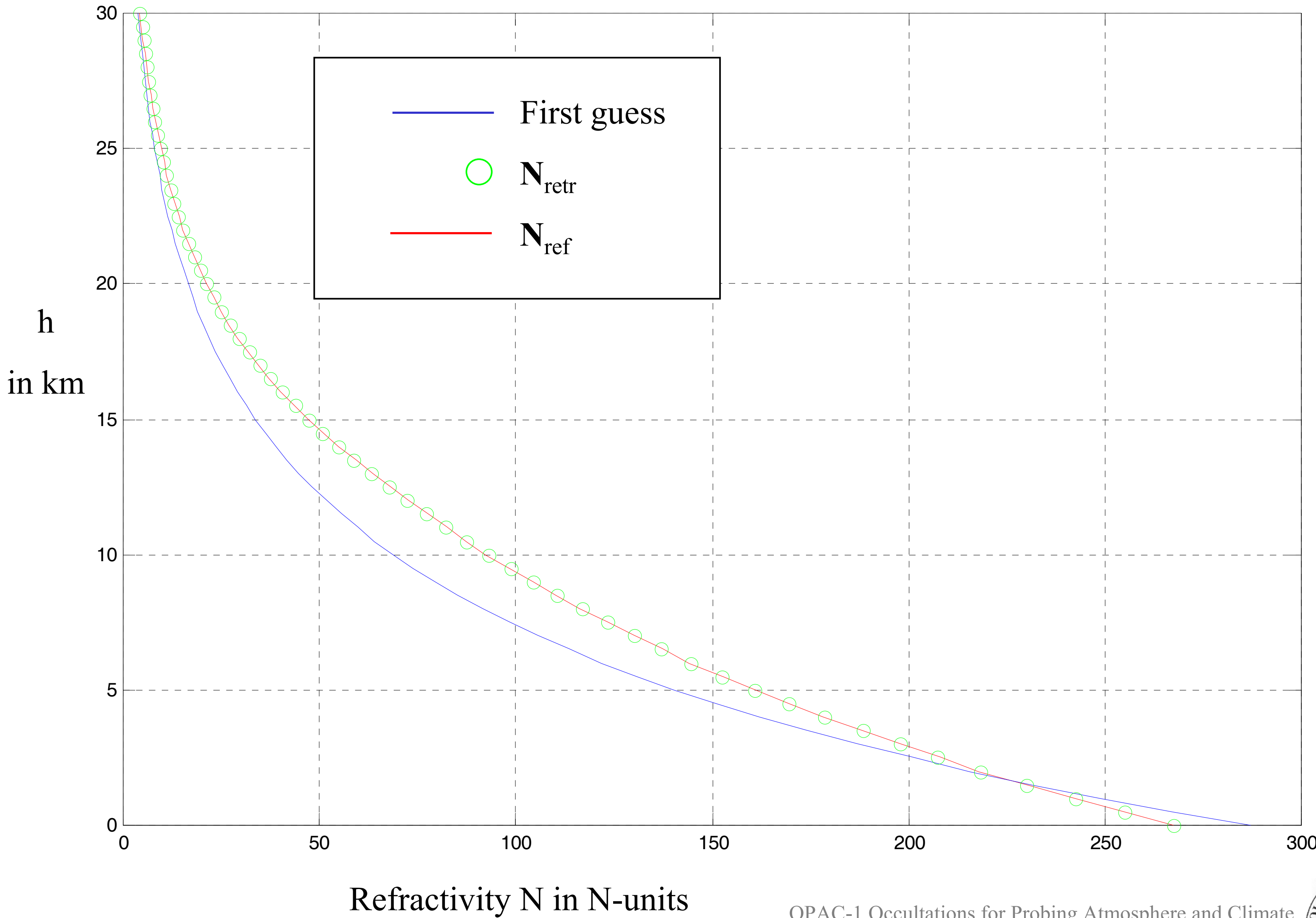
First guess profile: **Exponential model**



Results – refractivity profiles

July atmospheric model

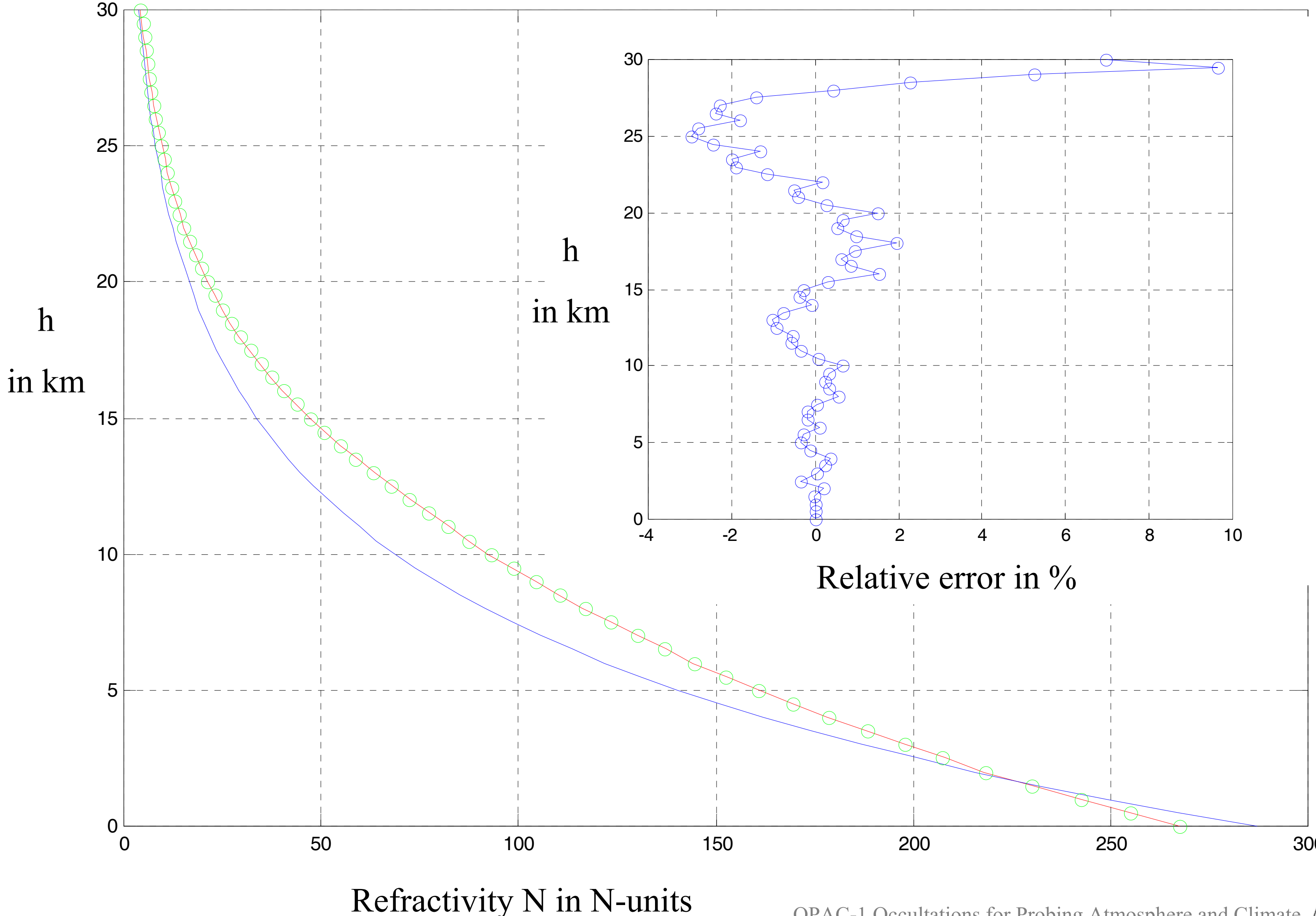
First guess profile: **Exponential model**



Results – refractivity profiles

July atmospheric model

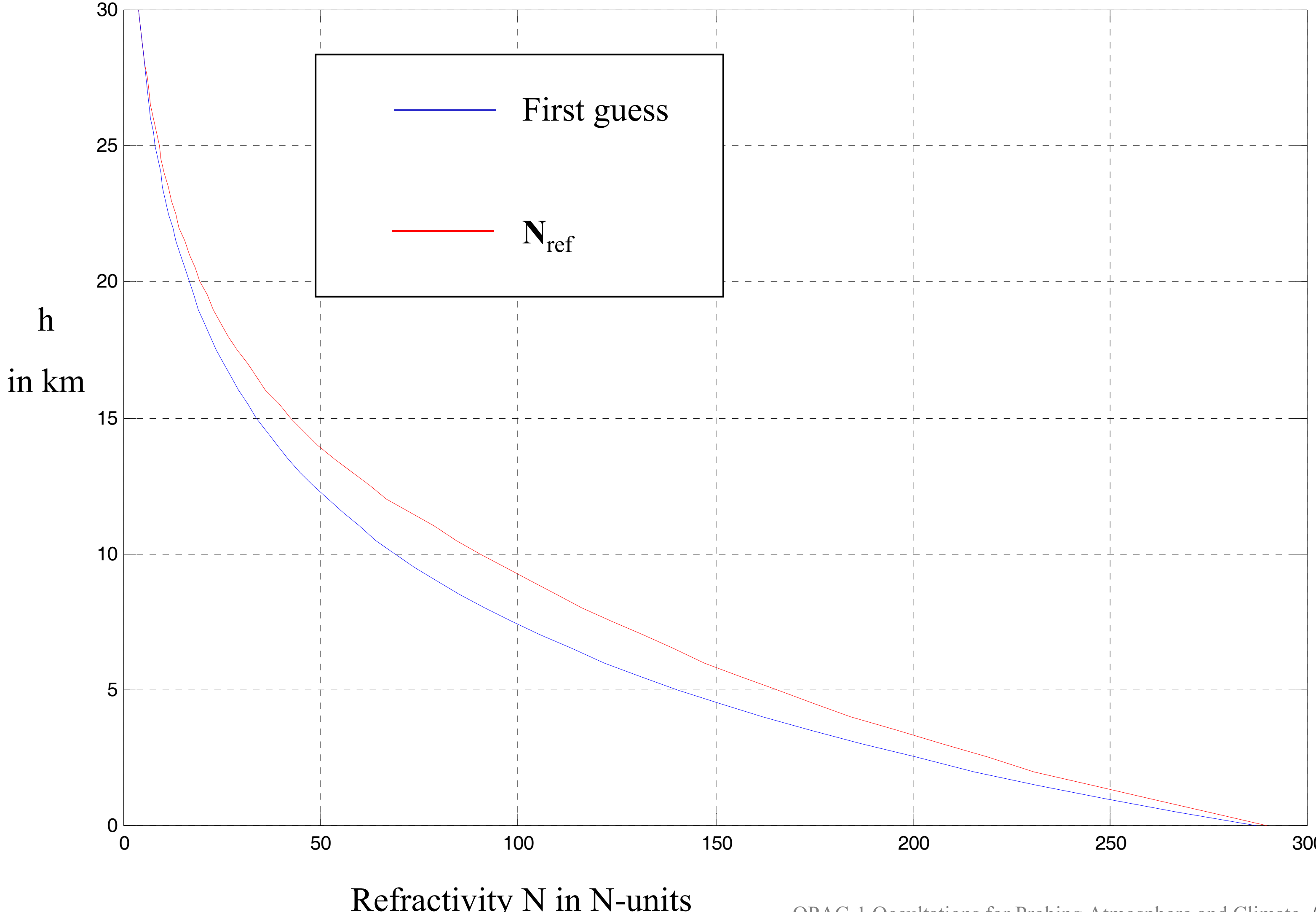
First guess profile: **Exponential model**



Results – refractivity profiles

January atmospheric model

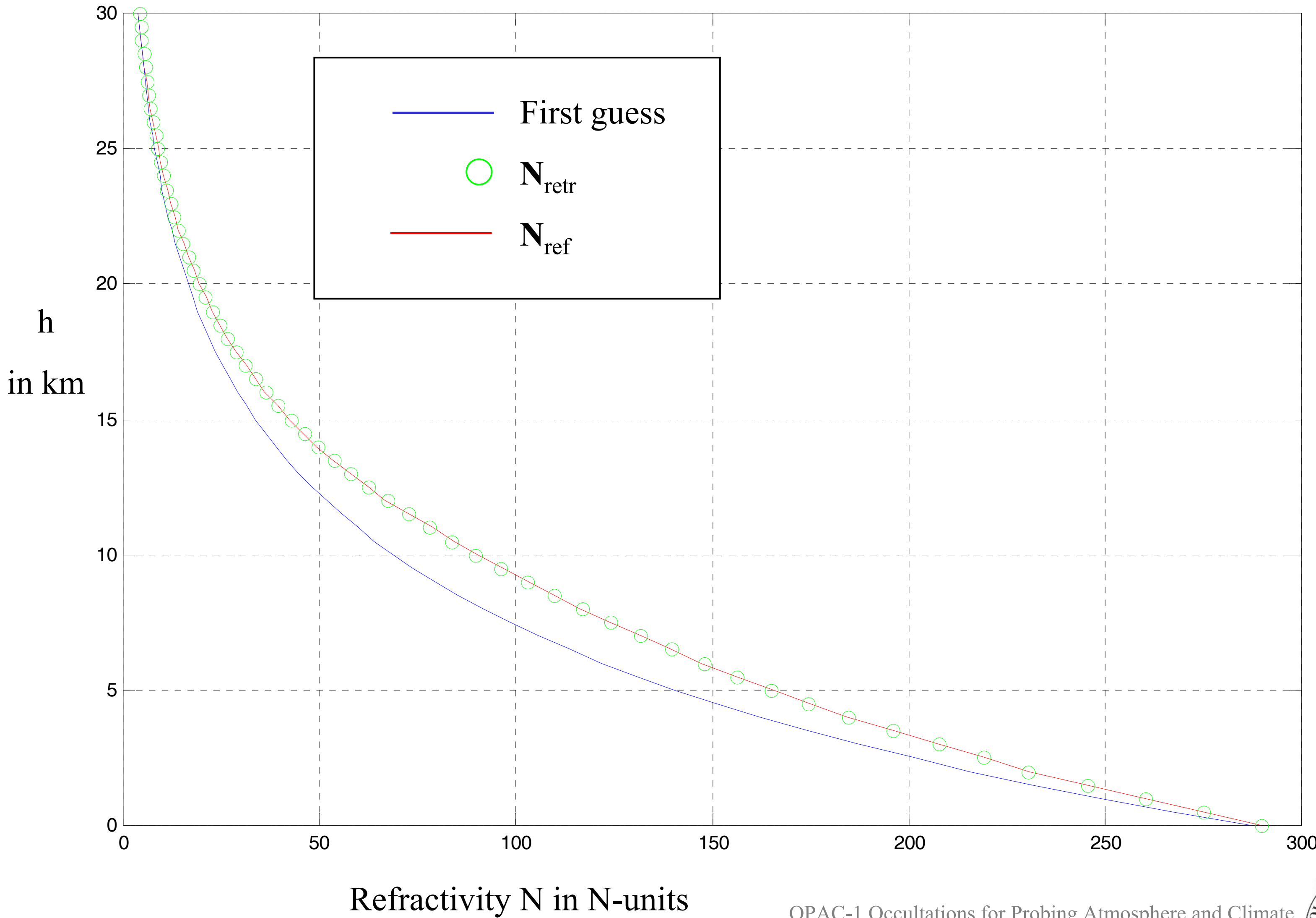
First guess profile: **Exponential model**



Results – refractivity profiles

January atmospheric model

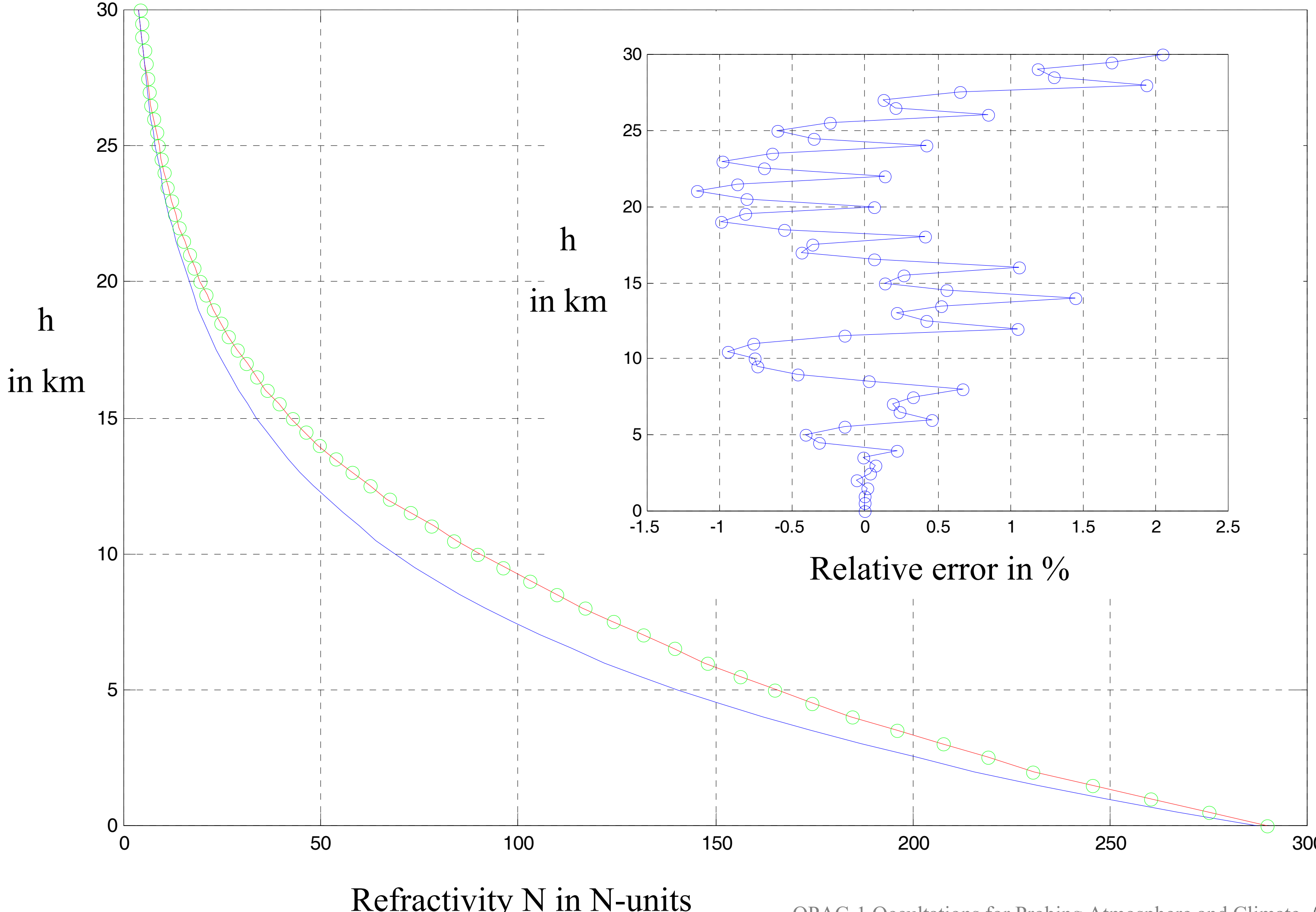
First guess profile: **Exponential model**



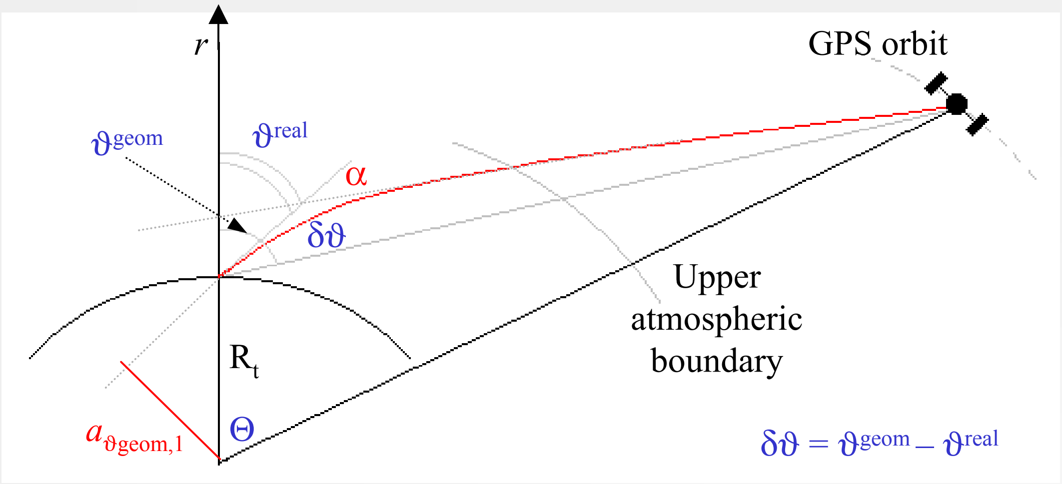
Results – refractivity profiles

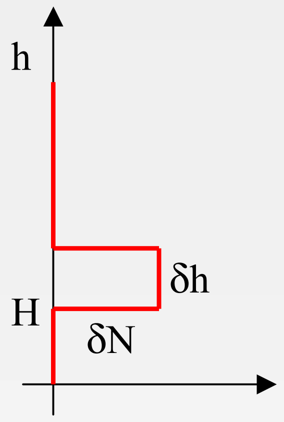
January atmospheric model

First guess profile: **Exponential model**

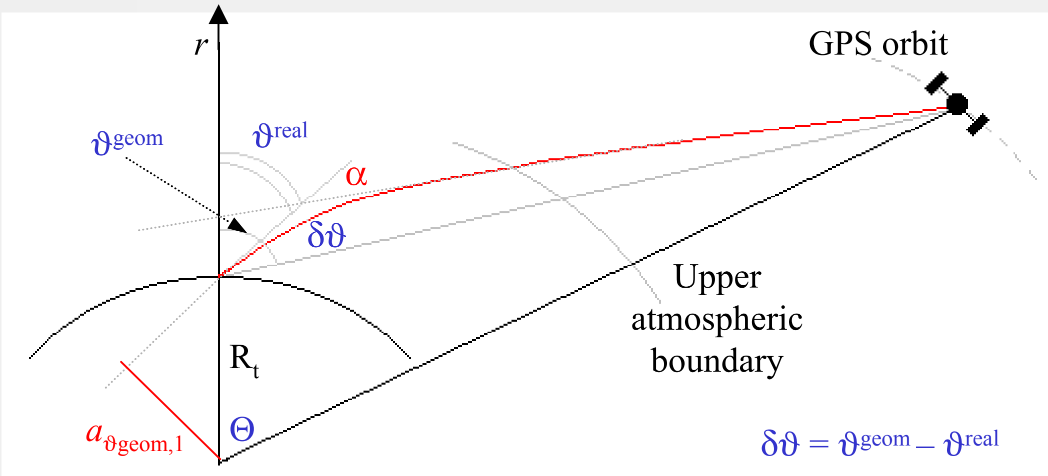


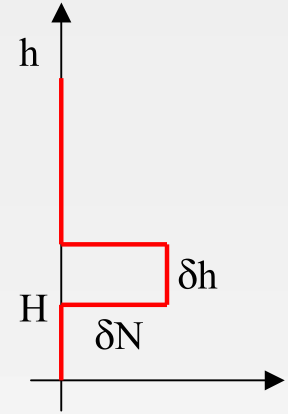
Sensitivity analysis: a simple approach





Sensitivity analysis: a simple approach



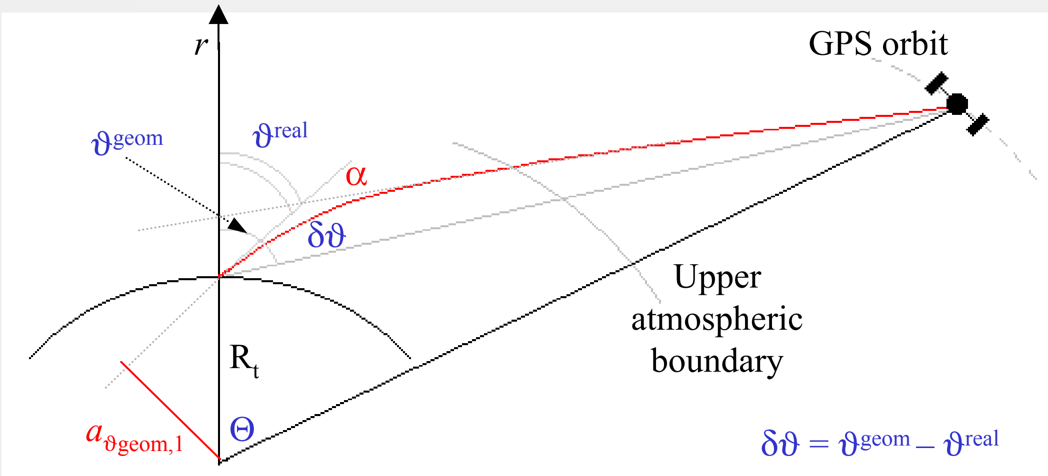


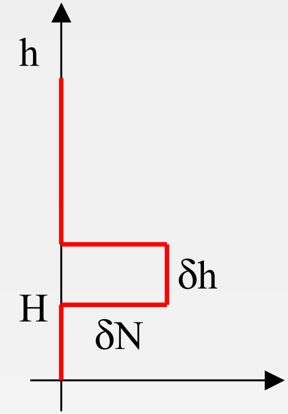
$$\Delta T_V(N(r),\vartheta_{real})$$

$$\Theta(N(r),\vartheta_{real}) = \Theta^{meas}$$

have been evaluated through the adoption of the atmospheric perturbation model

Sensitivity analysis: a simple approach





$\Delta T_V(N(r),\vartheta_{real})$

$\Theta(N(r),\vartheta_{real}) = \Theta^{meas}$

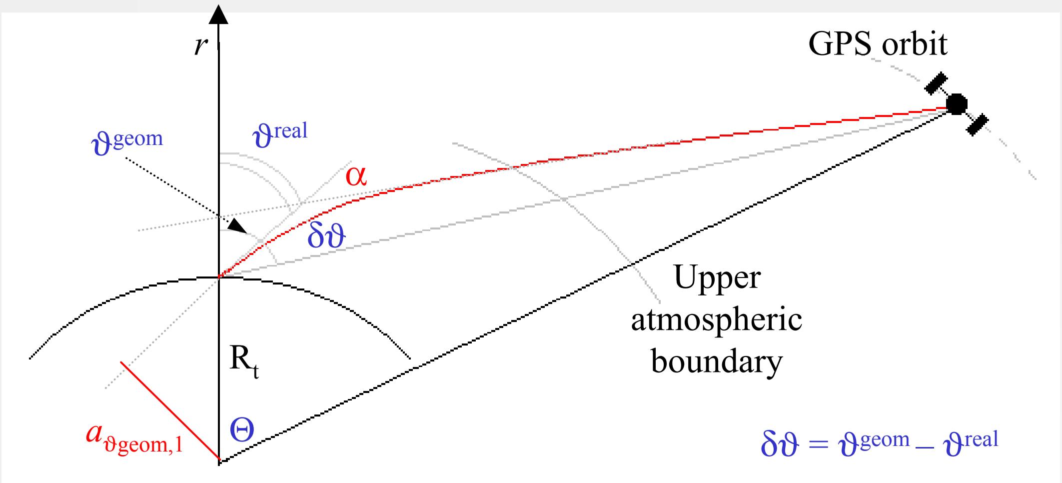
have been evaluated through the adoption of the atmospheric perturbation model

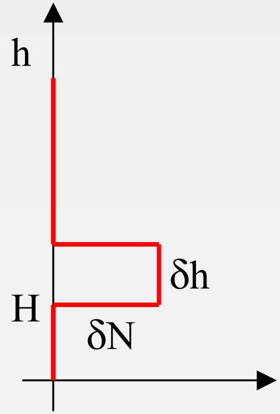
Function of ϑ_{geom}, H, δh and δN

$$\vartheta_{real} = \vartheta_{geom} - [f + g\,\Theta^{meas}]$$

$$\Delta T_V = \beta(\vartheta_{geom}, H, \delta h)\,\delta N$$

Sensitivity analysis: a simple approach





$\Delta T_V(N(r),\vartheta_{real})$

$\Theta(N(r),\vartheta_{real}) = \Theta^{meas}$

have been evaluated through the adoption of the atmospheric perturbation model

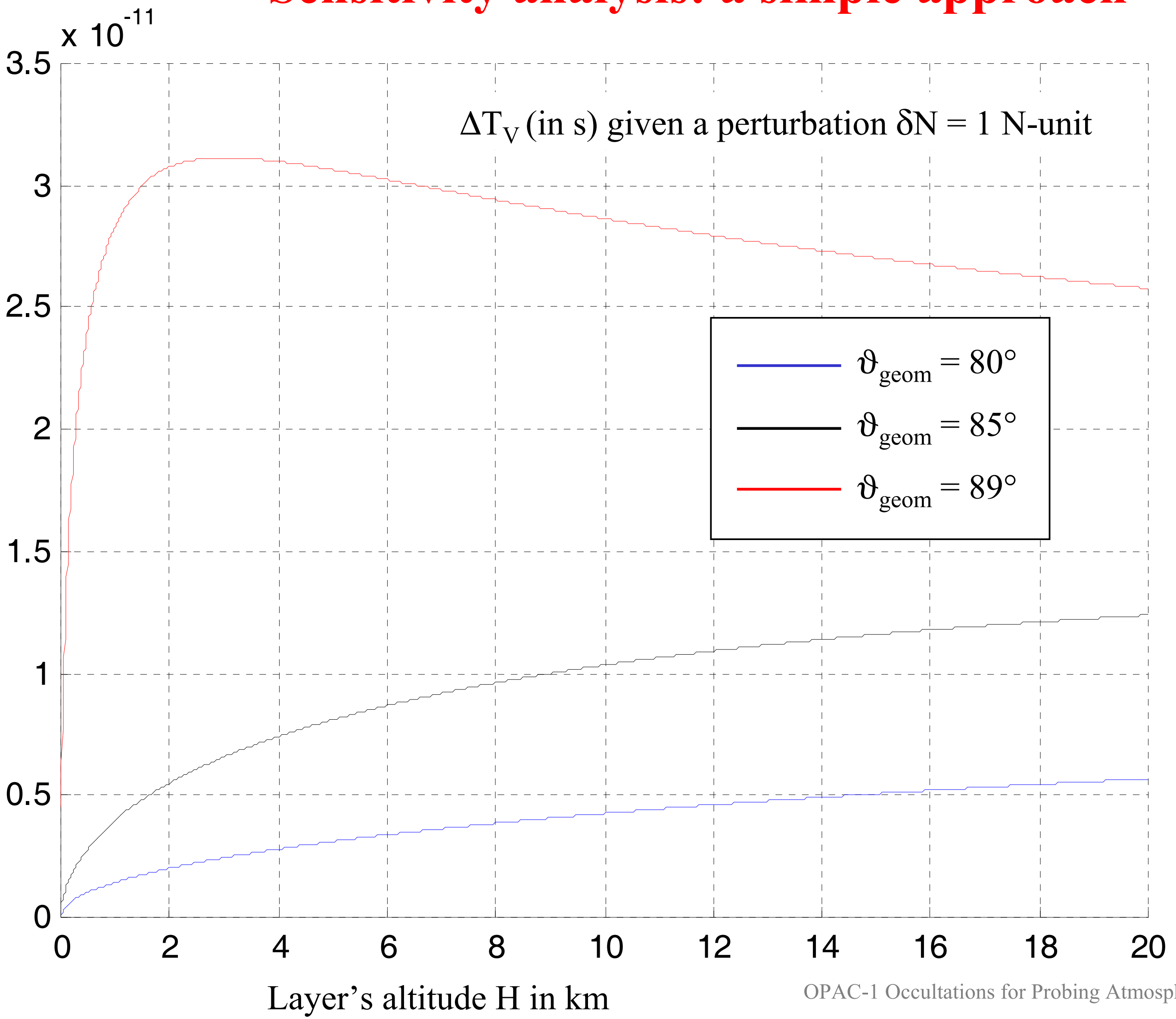
Function of ϑ_{geom}, H, δh and δN

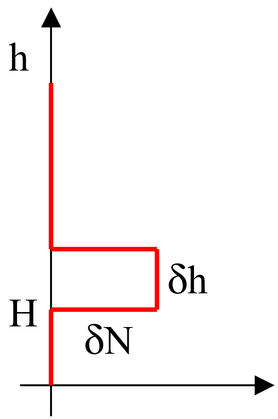
$$\vartheta_{real} = \vartheta_{geom} - [f + g\ \Theta^{meas}]$$

$$\Delta T_V = \beta(\vartheta_{geom}, H, \delta h)\ \delta N = \beta'(\vartheta_{geom}, H)\ \delta N$$

$$\Delta T_V \Leftrightarrow \delta N$$

Sensitivity analysis: a simple approach





Summary

- An inversion technique for the retrieval of refraction index profiles for well-mixed atmospheres is presented
- The methodology, so far verified only on a theoretical base, seems to be attractive
- and, thanks to a better horizontal resolution, its adoption could complement the good vertical resolution profiles obtainable through inversion of radio occulted GPS data

Outlook

In future:

- improvements on the optimization procedure
- sensitivity analysis
- experimental campaign

will have to be performed

Occultations for Probing Atmosphere and Climate

Sept. 16 – 20, Graz, Austria

An inversion approach for the retrieval of atmospheric refraction index profiles from ground-based GPS measurements

Riccardo Notarpietro, Marco Gabella, Giovanni Perona

Politecnico di Torino, Electronics Dept., Turin, Italy

Tel.: +39.011.5644623 Fax: +39.011.5647959 e-mail: rnp@polito.it