ABEL INTEGRAL INVERSION IN OCCULTATION MEASUREMENTS

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The occultation geometry under spherical symmetry assumption leads often to models describing by Abel integral equation. Analyzing general properties of the Abel transform, this work derives practical rules for discretization and for solution of the inverse problems, containing Abel-type integral equations. Two applications in remote sensing are considered: the vertical inversion in absorptive stellar occultation measurements and the reconstruction of air density from refractive angle measurements. In the case of continuous functions, it is shown that the vertical inversion problem is ill posed: small errors in measurements may cause errors of arbitrary size in retrieved quantities. The refractivity reconstruction problem is well posed: a noise in measurements is smoothed in the inversion.

In reality of finite number of measurements, the inverse problems can be made even-determined by discretization. The difficulties in discretization of the Abel-type integrals are the weak singularity at the lower limit and the upper limit initialization. Possible solutions to these problems are discussed as well as different discretization schemes.

The amplification of error is used as a criterion of ill- or well-posedness of the problems. Together with the averaging kernel, it also characterizes quality of the discretization schemes. For vertical inversion the standard onion peeling procedure and the onion peeling with polynomial profile model have almost the same quality: improved accuracy for polynomial scheme leads to degraded resolution.

For refractivity inversion, the dicretized inverse Abel transform is compared with the matrix inversion, formulated in two different ways. It is shown that the application of discretized Abel inverse transform gives the best approximation of the solution.

Necessity of regularization for the considered inverse problems is also discussed.